

## WRITTEN HW #8, DUE DEC 5 2011

Remember to write clearly and to justify all your claims in your solutions. Please staple your assignment before turning it in.

- (1) (10 points) An *automorphism* of a group  $G$  is an isomorphism of  $G$  into itself. For example, the identity function on  $G$  is an automorphism. How many automorphisms does a cyclic group of order  $n$  have?
- (2) (10 points) Show that 8 is a primitive root mod 557, but 16 is not. You may use a calculator/computer to compute powers mod  $n$  (you can use your own program!), but you should explain why you are calculating the powers you calculate and how they show what you want to show.
- (3) (10 points)  $p = 191$  is a prime. How many elements of  $U_{191}$  are
  - (a) squares?
  - (b) cubes?
  - (c) fifth powers?
  - (d) seventh powers?
- (4) (10 points)  $U_{36}$  is not cyclic, so  $U_{36}$  is not generated by any single element of  $U_{36}$ . However, find two elements which generate  $U_{36}$  (ie, two elements such that every element of  $U_{36}$  can be written as a product of powers of those two elements.)