Your Name Goes Here

Math 25

Homework 3

1. (a) Find the general solution to the congruence $35x \equiv 15 \pmod{120}$.
   (b) What does your work in the first part tell you about the solutions to $35x \equiv 15 \pmod{60}$?

2. Find the general solution to the congruence $35x \equiv 15 \pmod{121}$. Show all work including the steps to find a particular solution.

3. Find the least non-negative residue of $17^{208} \pmod{93}$.

4. Find all solutions to the system of congruences:
$$x \equiv 3 \pmod{17}, \qquad x \equiv 10 \pmod{16}, \qquad x \equiv 0 \pmod{15}.$$

5. A rafter of 17 turkeys have 12 nests, eleven of which contain the same number $m > 1$ of eggs, and the twelfth with 6 eggs. Dividing all the eggs into 17 piles produces piles each with the same number of eggs. What is the minimum number of eggs possible?

6. In this problem we show that the set of lattice points in the plane which are visible from the origin contain arbitrarily large square gaps. That is, we show that given any integer $k > 0$, there exists a lattice point (a, b) so that none of the lattice points
$$(a + r, b + s), 0 < r \leq k, 0 < s \leq k,$$
are visible from the origin.

   Suggested approach: Let $p_1, p_2, \ldots$ be the sequence of primes. Given $k > 0$, consider the $k \times k$ matrix whose first row is the first k primes, the second row is the next k primes, and so on. Let m_i be the product of the primes in the ith row, and M_j the product of the primes in the jth column.

   (a) Show that there are unique solutions modulo $M = m_1 m_2 \cdots m_k = M_1 M_2 \cdots M_k$ to the two systems of congruences:
$$\begin{array}{llll} x \equiv -1 & \bmod m_1 & y \equiv -1 & \pmod{M_1} \\ x \equiv -2 & \bmod m_2 & y \equiv -2 & \pmod{M_2} \\ \vdots & & \vdots & \\ x \equiv -k & \bmod m_k & y \equiv -k & \pmod{M_k} \end{array}$$

   Let a be the unique solution modulo M of the system with the m_is and b the solution to the system with the M_js.

   (b) Show that no lattice point of the form $(a+r, b+s)$ with $0 < r \leq k$ and $0 < s \leq k$ is visible from the origin.