Your Name Goes Here

Math 25

Homework 8

1. Determine congruence conditions which characterize all odd primes p for which $11 \in Q_p$.

2. Show that 3 is a primitive root of every Fermat number which is a prime greater than 3.

3. Let p be an odd prime, $a, b \in \mathbb{Z}$ with $p \nmid a$. Show that

$$\sum_{x=0}^{p-1} \left(\frac{ax+b}{p} \right) = 0.$$

4. Let $\mu(n)$ be the Möbius function defined for $n \geq 1$ by:

$$\mu(n) = \begin{cases} 1 & n = 1 \\ 0 & \text{if } n \text{ is not squarefree} \\ (-1)^t & \text{if } n = p_1 \cdots p_t \text{ is the product of } t \text{ distinct primes.} \end{cases}$$

   Your book defines $\mu(n)$ differently for which the definition above is Theorem 8.8. Let's prove that their definition follows from ours, that is: Prove that for all integers $n \geq 1$

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1. \end{cases}$$

   Hint: It might be useful first to show that μ is a multiplicative function.

5. Show that for all integers $n \geq 1$

$$\frac{n}{\phi(n)} = \sum_{d|n} \frac{\mu^2(d)}{\phi(d)},$$

   where μ is the Möbius function defined above.