

**Math 25**

## Homework 9 (Practice problems - not to hand in)

1. Show that  $\phi(n) = n \sum_{d|n} \frac{\mu(d)}{d}$ .
2. Let  $\Lambda(n) = \begin{cases} \ln p & n = p^e > 1 \text{ non-trivial prime power} \\ 0 & \text{otherwise} \end{cases}$   
where  $\ln x$  is the natural logarithm.
  - (a) Show that  $\sum_{d|n} \Lambda(d) = \ln n$ .
  - (b) Show that  $\Lambda(n) = \sum_{d|n} \ln(d)\mu(n/d) = - \sum_{d|n} \ln(d)\mu(d)$ .
3. Show that  $\sum_{d|n} \tau(d)^3 = \left( \sum_{d|n} \tau(d) \right)^2$ .