Clarification about Lemma (b):

If $\gcd(a,n)=1$ and $m \mid a$ and $m \mid b$, then

$$ax \equiv b \pmod{n} \quad \text{iff} \quad \left(\frac{a}{m}\right)x \equiv \frac{b}{m} \pmod{n}.$$

In the forwards direction:

$$3x \equiv 6 \pmod{7}$$

1. Check $\gcd(3,7)=1$.
2. Note $3 \mid 3$ and $3 \mid 6$.
3. Instead solve

$$x \equiv 2 \pmod{7}.$$

In the backwards direction INCORRECTLY:

$$19x \equiv 42 \pmod{50}$$

Multiply by 5:

$$95x \equiv 210 \pmod{50}$$

Here: $a=95$, $b=210$, $n=50$, and $m=5$.

Since $\gcd(95,50) \neq 1$, we cannot say

$$95x \equiv 210 \pmod{50}$$
$$\text{iff } 19x \equiv 42 \pmod{50}$$

In the backwards direction:

$$4x \equiv 13 \pmod{47}$$

1. Multiply both sides by 12

$$48x \equiv 156 \pmod{47}$$

2. Double check that $\gcd(48,47)=1$.

If so, then

$$4x \equiv 13 \pmod{47}$$
iff
$$48x \equiv 156 \pmod{47}.$$

Here: $a=48$, $b=156$, $n=47$, and $m=12$.

If not, then return to the original congruence.

In order for the backwards direction to be a valid move, choose m so that $\gcd(m,n)=1$.

Note: The statement of the lemma is correct, but its use requires some care.