Homework 7
(Quadratic reciprosity, arithmetic/multiplicative functions,
Mobius inversion, the Dirichlet product.)
Due Wednesday, November 13 at 11:30am in class.

Note: Be sure to justify your answers. No credit will be given for answers without work/justification. In addition, all written homework assignments should be neat and well-organized; **this assignment has only one part and can be submitted as a single packet**.

(1) Use the Law of Quadratic Reciprocity to determine if 73 is a quadratic residue mod 191.
(2) Use the Law of Quadratic Reciprocity to show that, for p prime,
$$[7] \in Q_p \text{ if and only if } p = 2 \text{ or } p \equiv \pm 1, \pm 3, \pm 9 \pmod{28}.$$
(3) An arithmetic function f is called *completely multiplicative* if $f(ab) = f(a)f(b)$ for all $a, b \in \mathbb{N}$. For example, for a fixed prime p, the Legendre symbol $\left(\frac{a}{p}\right)$ is a completely multiplicative function.
  (a) Determine if $\tau(n)$ is completely multiplicative. If it is, prove it. If not, find values of a and b for which $\tau(ab) \neq \tau(a)\tau(b)$.
  (b) Fix $m \in \mathbb{N}$. Let $f(n) = \gcd(m, n)$. Show that f is a multiplicative function. Is it completely multiplicative?
(4) Use the inductive definition of the Mobius function $\mu(n)$ to compute $\mu(150)$. You may not use the theorem giving the closed formula (Theorem 8.8), but you may use that $\mu(p) = -1$ for a prime p.
(5) (a) For each positive integer n, show that
$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0.$$
  (b) For any integer $n \geq 3$, show that
$$\sum_{k=1}^{n} \mu(k!) = 1.$$