Homework 8

(Fermat's Last Theorem, Pythagorean triples, sums of squares.)

Due Monday, November 18 at 11:30am in class.

Note: Be sure to justify your answers. No credit will be given for answers without work/justification. In addition, all written homework assignments should be neat and well-organized; **this assignment has only one part and can be submitted as a single packet**.

(1) Show that $(25, 312, 313)$ is a primitive Pythagorean triple.

(2) Show that there is not Pythagorean triple (a, b, c) such that $a = 2b$. *Do not use the fact that $\sqrt{5}$ is irrational.* Conclude that $\sqrt{5}$ is irrational.

(3) (a) Show that if n is a sum of three perfect squares, then $n \not\equiv 7 \pmod 8$.
(Hint: Write $n = a^2 + b^2 + c^2$ and consider $n \pmod 8$. What values are perfect squares mod 8?)

(b) Show that if n is a sum of three perfect squares and n is divisible by 4, then $\frac{n}{4}$ is also a sum of three perfect squares.
(Hint: Write $n = a^2 + b^2 + c^2$ and consider $n \pmod 4$. What values are perfect squares mod 4?)

(c) Show that if $n = 4^e(8k + 7)$, then n is not a sum of three perfect squares.
(Hint: Proceed by contradiction.)