Final Examination
Math 29, Spring 2003

Study Guide: The final is cumulative, though weighted toward the second half of the course. You should therefore review material covered in the first half of the course, incluing: basic definitions of computability, the operations of computable functions, the coding whereby instructions and programs are coded as natural numbers, Church's Thesis and evidence for it given in class, the exact statements of the $s-m-n$ and Universal Program theorems, the existence of noncomputable functions, and what all this has to do with decidable and partially decidable predicates.

Since the mid term, we have discussed Chapter 7, the structure of recursive and recursively enumerable sets, the first two sections of Chapter 8, (which give an informal glimpse of the Gödel Incompleteness Theorem), and sections 4 and 5 of Chapter 9, concerning the Turing degrees. Some review problems for Chapter 9 are: page 173, 6, 10 and page 181, 1,4. For chapter 7, look at the homework problems assigned when we covered it, and for Chapter 8, read over the two sections.

Here are some other questions that you might want to think about before the exam. They are not in every case an exact forshadowings of what will be on the test, but they are indicative of the kind of question, etc.

(1) What is a universal program? Carefully state the theorem that says they are computable.

(2) (a) Let ${}^nW_e = \{x : \phi_c(x) = n\}$. Show that for all n, ${}^nW_e$ is r.e.
(b) Does the enumeration ${}^0W_e, {}^1W_e, {}^2W_e, {}^3W_e, \ldots$ include all the r.e. sets?

(3) Prove or disprove (by producing and defending an explicit counterexample) the following statement: for every $A \subseteq \mathbb{N}$, either A or $\bar{A}$ is r.e.

(4) Show that for any set A, $A <_T K^A$. (*Hint:* You need to show that the ordering is strict.

(5) Recall that if A and B are subsets of $\mathbb{N}$, then

$$A \oplus B = \{x \mid x = 2a, \text{for some a} \in A \text{ or } x = 2b+1, \text{for some b} \in B\}$$

Show that $deg_T(A \oplus B)$ is the unique least upper bound of $deg_T A$ and $deg_T B$. (*Hint:* To show that $deg_T(A \oplus B)$ is the minimal degree above both A and B, show that if a set C in $deg_T(A \oplus B)$, then it is either in $deg_T A$ or $deg_T B$.)