Math 29: Homework 4

Due Wednesday, April 26

1. ($B(n)$ for $n \leq 10$) Give a lower bound on $B(n)$ for each $n \leq 10$. (Note that Exercise 5.4.4 says that 39 is possible for $n = 10$; you are not required to find this for full credit — but it would be nice!)

2. Find the flaw in this argument:

   > To compute $B(n)$, the output of the busy beaver function on input n, list the finitely many URM programs of length n and calculate each one on input 0. Take the largest value you see. This is an algorithm to compute $B(n)$ and hence, by Church's thesis, B is URM-computable.

3. (creative sets are not computable) Exercise 6.1.2.

4. Show that the relation "$\phi_x = \phi_y$" is undecidable.

5. (a Diophantine relation) Express $|m - n| = k$ as a Diophantine relation. That is, find a polynomial $p(m, n, k, y_1, \ldots, y_l)$, for some $l \geq 0$, such that

$$|m - n| = k \quad \Leftrightarrow \quad \exists y_1 \in \mathbb{N} \cdots \exists y_l \in \mathbb{N} \;\; p(m, n, k, y_1, \ldots, y_l) = 0.$$