Math 29: Homework 5

Due Wednesday, May 10

1. (c.e. $\Leftrightarrow$ below K_0) Exercise 7.1.6.

2. *This question is about <u>all</u> subsets of* $\mathbb{N}$. *Computability plays no part here.* For subsets A and B of $\mathbb{N}$, let $A \subseteq^* B$ if $A \setminus B$ is finite. Let $A \equiv_{fd} B$ if $A \subseteq^* B$ and $B \subseteq^* A$. Write $[A] = \{X \subseteq \mathbb{N} : A \equiv_{fd} X\}$. Let $[A] \leq_{fd} [B]$ if and only if $A \subseteq^* B$.

   (a) Show that $\subseteq^*$ is reflexive and transitive.

   (b) Show that $\equiv_{fd}$ is an equivalence relation.

   (c) Is $\{[A] : A \subseteq \mathbb{N}\}$ a Boolean algebra? Justify your answer.

   (d) Show that for any $[A] \neq [\mathbb{N}]$ there is a set B with $[A] <_{fd} [B] <_{fd} [\mathbb{N}]$.

3. (computable sets are bottom) Exercise 7.2.14.

4. (lattice of c.e. degrees) Exercise 7.2.18.

5. (join in m-degree) Exercise 7.2.19.