

## Math 29: Final - Take-Home Portion

Due June 7th - 11:59 PM

For each of the following questions, provide a complete, clear solution. Remember to make it obvious which problem you are solving in each solution. Virtual submissions are due by midnight on the due date, either via Gradescope or email. You are only permitted to use the lecture notes, the course textbook, and my help as a resource - no collaboration, internet, or other sources.

1. Are there sets  $A$  and  $B$  such that  $(A \oplus B)' \not\equiv_T A' \oplus B'$ ? Justify your answer.
2. Give a set  $X$  such that  $\emptyset' <_T X <_T \emptyset''$ , i.e. a set strictly between the first and second jump. Justify your answer.
3. Is  $\{e : \varphi_e(3) \downarrow\}$  Turing equivalent to  $\emptyset'$ ? Justify your answer.
4. Prove that  $\emptyset''$  can compute  $\{e : W_e \text{ is finite}\}$ .
5. Let  $X$  and  $Y$  be such that  $X \perp_T Y$ . Prove that there is an  $e$  such that  $\Phi_e^X = \{e\}$  and  $\Phi_e^Y = \omega \setminus \{e\}$ .
6. Give a computable tree  $T \subseteq \omega^{<\omega}$  (i.e. a computable set of finite strings of natural numbers which is closed under initial segments) which contains exactly one path  $X$ , and such that  $X$  computes  $\emptyset'$ .