Math 29: Homework 1

Due April 6th

For each of the following questions, provide a complete, clear solution. Remember to make it obvious which problem you are solving in each solution. Virtual submissions are due by midnight on the due date, either via Gradescope or email. Physical solutions are due in class on the due date.

1. (a) Let $z : \mathbb{Z} \to \omega$ be the coding map defined in Wednesday's class. Prove that z is bijective.

   (b) Let X be the set of finite subsets of the natural numbers and let $f : X \to \omega$ be the coding map defined in Wednesday's class. Prove that f is bijective.

2. Write down a polynomial coding function which gives a bijection between ω^4 and ω. Justify why it is a bijection. Hint: Use the coding function given for ω^2 and rely on the fact that we know it is a bijection.

3. Let $p^k : \omega^k \to \omega$ be coding functions which assign a code to each k-tuple for each k. Let X be the set of all tuples of natural numbers of size at least 2. Give a bijection $g : X \to \omega$ in terms of the p^k's.

4. The rational numbers, $\mathbb{Q}$, are $\{\frac{p}{q} : p \in \mathbb{Z},\ q > 0\}$. Provide a bijection between $\mathbb{Q}$ and ω.

5. Prove that multiplication is computable by providing a register machine.

6. Prove that exponentiation is computable by providing a register machine.

7. Give a register machine which computes the characteristic function of the odd numbers.

8. Give a register machine which diverges if there is an odd number in the first register and converges otherwise.