Math 29: Homework 2

Due April 13th

For each of the following questions, provide a complete, clear solution. Remember to make it obvious which problem you are solving in each solution. Virtual submissions are due by midnight on the due date, either via Gradescope or email. Physical solutions are due in class on the due date.

1. Show that the factorial function $n!$, where $0! = 1$ and $n! = n * (n-1)!$, is computable by building a register machine.
2. Show that the Fibonacci function, where $f(0) = f(1) = 1$ and $f(n+2) = f(n+1) + f(n)$, is computable by building a register machine.
3. Show that the set of multiples of 8 is computable by building a Turing machine.
4. Show that the set of powers of 4 is computable by building a Turing machine.
5. Write a register machine that computes the modulus function, i.e. the function $\% : \omega^2 \to \omega$ such that $\%(x, y)$ is the remainder of x after division by y.
6. Describe, in words or diagrams, how you could represent the tape for a Turing machine inside of a single register using ternary. Also describe how you could read or write elements of the tape with a register machine. (You may reference the fact that we have built register machines which can compute addition, multiplication, division, exponentiation, and modulus.)
7. Describe, in words or diagrams, a register machine which can simulate a single state of a Turing machine. In other words, the module checks to see if it is in a given state, and moves on if it is not. If it is in the given state, it should read something from the input tape and, depending on the value, perform some action and enters a different state. You should assume that you have a single register containing the tape as per the previous question, a single register containing the location of the Turing machine head, and that each state has its own register.