

Math 29: Homework 3

Due April 20th

For each of the following questions, provide a complete, clear solution. Remember to make it obvious which problem you are solving in each solution. Virtual submissions are due by midnight on the due date, either via Gradescope or email. Physical solutions are due in class on the due date.

1. Let N_k be the set of numbers not divisible by the k -th prime p_k . Show that $\{\chi_{N_k}\}_{k \in \omega}$ is uniformly computable.
2. Is there a fastest growing total computable function, i.e. a function $f(n) : \omega \rightarrow \omega$ such that $f(n+1) - f(n) \geq \varphi_e(n+1) - \varphi_e(n)$ for all e such that φ_e is total and all n ? Prove that your answer is correct.
3. Use a diagonalization argument to show that there is no uniform listing of all characteristic functions of the computable sets.
4. Prove that a set X is computable if and only if it and its complement (that is, $\{n : n \notin X\}$) are c.e.
5. Prove that every c.e. set contains a computable subset.
6. Prove that every total computable function has infinitely many fixed points.
7. Prove that there is an e with $W_e = \{e, 2e, 3e, 4e, \dots\}$.