

Math 29: Homework 4

Due April 27th

For each of the following questions, provide a complete, clear solution. Remember to make it obvious which problem you are solving in each solution. Virtual submissions are due by midnight on the due date, either via Gradescope or email. Physical solutions are due in class on the due date.

1. The **join** of two sets A and B , denoted by $A \oplus B$, is defined via

$$A \oplus B = \{2n : n \in A\} \sqcup \{2n + 1 : n \in B\}$$

In other words, the elements of A are coded along the evens and the elements of B are coded along the odds. Prove that $A \oplus B$ is computable if and only if both A and B are.

2. Prove that $A \oplus B$ is c.e. if and only if A and B are.
3. Does Lemma 1 from the Noncomputability lecture hold true if we remove the word total? That is, is f partial computable if and only if its graph is a computable set? Justify your answer.
4. Recall that W_e is $\text{dom}(\varphi_e)$, and that X is c.e. if $X = W_e$ for some e . Show that it is equivalent to define the c.e. sets as those that are either finite or equal to the range of a total, computable, injective function $f : \omega \rightarrow \omega$.
5. Prove that a c.e. set is computable if and only if it is the range of an increasing, total, computable function.
6. Prove that K (the halting set) is **not** an index set.
7. Is H a creative set? Justify your answer.
8. Show that if P is productive, then it contains an infinite c.e. set.