Math 29: Homework 5

Due May 11th

For each of the following questions, provide a complete, clear solution. Remember to make it obvious which problem you are solving in each solution. Virtual submissions are due by midnight on the due date, either via Gradescope or email. Physical solutions are due in class on the due date.

1. Prove that if A and B are computable, infinite, and coinfinite, then $A \equiv B$.

2. Can any of the three modifiers in the above problem be removed for it to be true in complete generality? (That is, if one or more of the words is removed, for example computable, is $A \equiv B$ for any A and B satisfying the remaining ones?)Justify your answer.

3. Show that if C is a computable set, then $C \leq_m X$ for any set X which is nonempty and has nonempty complement.

4. Let A be an infinite computable set. Is there an immune set I such that $A \leq_1 I$? Justify your answer.

5. Let B be an infinite c.e. set. Is there an immune set I such that $B \leq_1 I$? Justify your answer.

6. Is there a simple index set? Justify your answer.

7. Please list the part of class (lecture, homework, textbook, etc.) that you find most beneficial for your learning.

8. Please list one thing that you think could be improved about the course.