Math 29: Homework 7

Due May 25th

For each of the following questions, provide a complete, clear solution. Remember to make it obvious which problem you are solving in each solution. Virtual submissions are due by midnight on the due date, either via Gradescope or email. Physical solutions are due in class on the due date.

1. Prove that $A' \leq_1 B'$ implies $A \leq_T B$.
2. Define a set X such that X computes $\emptyset^{(n)}$ for all n uniformly, i.e. there is an e such that $\Phi_e^X(n,k) = \chi_{\emptyset^{(n)}}(k)$. Justify your answer.
3. Give a limit computable function which does not have c.e. degree.
4. Prove that there is a fixed point-free (i.e. $W_x \neq W_{f(x)}$ for all x) function $f \leq_T X$ if and only if there is $g \leq_T X$ such that g is fixed point-free in the previous sense, i.e. $\varphi_{g(x)} \neq \varphi_x$.
5. Prove that, for all n and $f : \omega \to \omega$, there is a computable function $g : \omega^{n+1} \to \omega$ such that
$$f(x) = \lim_{s_0 \to \infty} \lim_{s_1 \to \infty} \cdots \lim_{s_{n-1} \to \infty} g(x, s_0, s_1, \ldots, s_{n-1})$$
if and only if $f \leq_T \emptyset^{(n)}$.
6. Give an example of a set X such that $X \perp_T \emptyset^{(n)}$ for all $n > 1$. (Hint: We are not required to perform (priority) constructions computably.)
7. Recall that $X =^* Y$ if X and Y agree on all but finitely many numbers. Show that there are sequences of sets $\{A_n\}_{n\in\omega}$ and $\{B_n\}_{n\in\omega}$ such that $A_n =^* B_n$ for all n, but $\oplus_{n\in\omega} A_n \not\equiv_T \oplus_{n\in\omega} B_n$.