Math 2 Winter 2006 Quiz 3

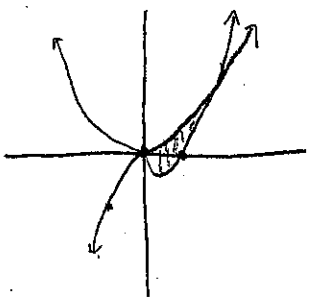
Name: Key Section:

1. Find $\frac{d}{dx}\left(\int_0^{\frac{\pi}{2}x^2} \cos^3(t)dt\right)$.

$$\cos^3\left(\tfrac{\pi}{2}x^2\right) \cdot \frac{d}{dx}\left(\tfrac{\pi}{2}x^2\right)$$

$$\pi x \cdot \cos^3\left(\tfrac{\pi}{2}x^2\right)$$

2. Find the area of the region bounded by the curves $y = x^3 - x^2$ and $y = 3x^2$. (Start by sketching the region.)



$x^3 - x^2 = 3x^2 \Rightarrow x^3 - 4x^2 = 0$

$x^2(x-4) = 0$

intersect @ $x = 0, x = 4$

$$\int_0^4 \left(3x^2 - (x^3 - x^2)\right)dx$$

$$= \int_0^4 (-x^3 + 4x^2)\,dx = -\tfrac{1}{4}x^4 + \tfrac{4}{3}x^3 \Big|_0^4$$

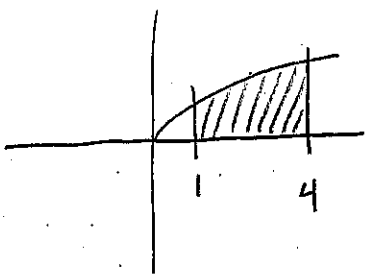
$$= -\tfrac{1}{4}\cdot 4^4 + \tfrac{4}{3}\cdot 4^3$$

$$= -64 + \tfrac{4}{3}\cdot 64$$

$$= \tfrac{1}{3}\cdot 64$$

$21\frac{1}{3}$

3. Find the volume of the solid of revolution created by rotating the region bounded by $y = 3\sqrt{x}$, $x = 1$, $x = 4$ and $y = 0$ about the x-axis. (Start by sketching the region.)



$$\int_1^4 \pi (3\sqrt{x})^2 \, dx = 9\pi \int_1^4 x \, dx$$

$$= 9\pi \cdot \frac{1}{2} x^2 \Big|_1^4$$

$$= \frac{9}{2}\pi (16 - 1)$$

$$= \frac{9 \cdot 15}{2} \pi$$

$\frac{135}{2}\pi$