Math 2 – Practice Midterm 2

1. Sketch the region bounded by the graphs of the equations $y = -x^2$ and $y = x^2 - 8$. Find the area by integrating with respect to x.

2. Find the area of the region between the graphs of the equations $y = \cos\left(\frac{1}{2}x\right)$ and $y = \sin(x)$ from $x = \pi/3$ to $x = \pi$. [Note: The graphs do not intersect on the interval $(\pi/3, \pi)$.]

3. Consider the region R bounded by the graphs of the equations $y = (x-5)^2$ and $y = 4$.
   (a) Find the volume of the solid generated by revolving the region R about the x-axis.
   (b) Find the volume of the solid generated by revolving the region R about the line $y = 4$.

4. Consider the region R bounded by the graphs of the equations $y = \sqrt{x}$, $x = 0$ and $y = 2$.
   (a) Find the volume of the solid generated by revolving the region R about the y-axis.
   (b) Find the volume of the solid generated by revolving the region R about the line $x = 4$.
   (c) Find the volume of the solid generated by revolving the region R about the line $x = -1$.

5. A solid has as its base the region in the xy-plane bounded by the graphs of $y = x$ and $y = -x^2$. If every cross section by a plane perpendicular to the x-axis is a rectangle with height equal to one half the length of its base, find the volume of the solid.

6. Using a definite integral, derive a formula for the volume of right circular cone with height h and radius r.

7. Set up an integral that can be solved to find the arc length of the curve $y = \cos^8(x^4)$ from the point $(0, 1)$ to $\left(\sqrt[4]{\pi/2}, 0\right)$.

8. Find the length of the graph of $y = 2x^{3/2}$ from the point $(1, 2)$ to the point $(4, 16)$.

9. Using a definite integral, derive a formula for the surface area of a sphere of radius r.