Math 2 Due: January 18, 2008

Written Problem 2

My car, a '92 VW Cabriolet convertible, is called Little Red. Now, I think she's pretty awesome, but some other people are not so convinced. One guy in particular says she can't go fast enough. "It must take days to get that car up to freeway speed." Well, I've never been able to test this theory, since I haven't found a place where I can go from a complete stop and accelerate to freeway speeds. I do know, however, if I push the gas pedal all the way down leaving a stop light, I can reach 25 mph in 4 seconds. If I assume Little Red accelerates at a constant rate, how fast can she go from 0 to 60 mph? Although 0 to 60 is an interesting statistic to report, it doesn't directly apply to any of the driving I do. Suppose by the time I reach the end of the on-ramp, I have reached 35 mph. How much longer does it take for me to reach the speed of traffic, say 70 mph? How far onto the freeway am I by that time?

(Make sure you include all the units in your calculations and write up your solutions clearly and neatly. Use complete sentences and correct grammar to explain your process and reasoning.)

We will be using same ideas from rectilinear motion, namely that

$\underline{a(t) = v'(t) = s''(t)}$ where a, v, and s represent acceleration, velocity and position respectively. For this set-up we know acceleration is constant, so $a(t) = a$, and also that $v(0) = 0$ mph & $v(4) = 25$ mph, where the t-values (0 & 4) are given in seconds. To be able to use this information we need an expression for velocity, which we find by taking the antiderivative of the acceleration function:

$$a(t) = a \Rightarrow v(t) = at + c.$$

We have two unknowns, a & c, and two conditions to satisfy, $v(0)=0$ & $v(4)=25$. This gives:

$$v(0) = 0 \cdot a + c \qquad v(4) = 4 \cdot a + 0$$

$$0 = c \qquad 25 = 4a$$

So $v(t) = (6.25 \frac{mph}{s})t$

$$a = \frac{25}{4} \frac{mph}{s.} = 6.25 \frac{mph}{s}$$

Miles per hour per second is a strange unit, so we must be careful to keep track of it. Now, the first question is how long does it take to reach 60mph. We use our velocity function and solve for t:

$$v(t) = (6.25 \frac{mph}{s})(t) = 60\text{ mph}$$

$$t = \frac{60\text{ mph}}{6.25 \frac{mph}{s}} = \frac{60}{6.25} s = 9.6s.$$

So it takes 9.6 seconds for Little Red to go from 0 to 60 mph.

To know the time it takes to get from 35mph to 70mph we simply find the times, t, when each velocity occurs and then subtract to find the difference:

$$v(t) = 35\text{mph} \qquad v(t) = 70\text{mph}$$

$$(6.25 \frac{mph}{s})(t) = 35\text{mph} \qquad (6.25 \frac{mph}{s}) t = 70\text{ mph}$$

$$t = 5.6s. \qquad t = 11.2s$$

So, it takes Little Red (11.2−5.6) 5.6 seconds to get from 35 mph to 70 mph.

The question of distance gets slighty more complicated. We find the position function by taking the antiderivative of velocity:

$$v(t) = \left(6.25\,\frac{mph}{s}\right)t \Rightarrow s(t) = \left(6.25\,\frac{mph}{s}\right)\frac{t^2}{2} + C.$$

Since we start out at time $t=0$ covering no distance, $s(0)=0 \Rightarrow c=0$. So,

$$s(t) = \left(6.25\,\frac{mph}{s}\right)\frac{t^2}{2}.$$

We find the distance traveled before the car reaches 35 mph, at $t=5.6s$, and the total distance traveled until the car reaches 70 mph, $t=11.2s$. The difference of these distances will give the distance traveled between 35 mph & 70 mph.

$$s(5.6s) = \left(6.25\,\frac{mph}{s}\right)\frac{(5.6s)^2}{2}$$

$$= 98\,\frac{mi}{hr\cdot s}\cdot s^2$$

$$s(11.2s) = \left(6.25\,\frac{mph}{s}\right)\frac{(11.2s)^2}{2}$$

$$= 392\,\frac{mi}{hr\cdot s}\cdot s^2$$

So the distance traveled on the freeway is $(392-98)\frac{mi}{hr \cdot s} s^2 = 294 \frac{mi \cdot s}{hr}$.

Now mile seconds per hour is not a very understandable distance. We convert:

$$\frac{294 \text{ mi} \cdot \cancel{s}}{1 \cancel{hr}} \cdot \left(\frac{1 \cancel{hr}}{3600 \cancel{s}}\right) = .08167 \text{ mi},$$

which is still not very understandable, so

$$.08167 \text{ mi} \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) = 431.2 \text{ ft}.$$

So, Little Red only needs 431.2 ft to reach a reasonable freeway speed.