Math 2, Winter 2016

Daily Homework #10 — Solutions

3.9.14. *Find the most general antiderivative of* $3\cos(t) - 4\sin(t)$.

Solution. An antiderivative of $\cos(t)$ is $\sin(t)$, and an antiderivative of $\sin(t)$ is $-\cos(t)$, so the most general antiderivative of $3\cos(t) - 4\sin(t)$ is $3\sin(t) + 4\cos(t) + C$.

3.9.20. *Find the antiderivative* F *of* $f(x) = x + 2\sin(x)$ *that satisfies* $F(0) = -6$.

Solution. An antiderivative of x is $\frac{1}{2}x^2$, and an antiderivative of $\sin(x)$ is $-\cos(x)$, so the *general* antiderivative of f is

$$F(x) = \tfrac{1}{2}x^2 - 2\cos(x) + C.$$

We find C by setting $x = 0$ and substituting $F(0) = -6$:

$$F(0) = \tfrac{1}{2}(0^2) - 2\cos(0) + C \quad \Rightarrow \quad -6 = -2 + C \quad \Rightarrow \quad C = -4.$$

Therefore, $F(x) = \frac{1}{2}x^2 - 2\cos(x) - 4$.

3.9.33. *Find* f*, given that* $f''(x) = -2 + 12x - 12x^2$ *and* $f(0) = 4$ *and* $f'(0) = 12$.

Solution. If $f''(x) = -2 + 12x - 12x^2$, then the antiderivative is

$$f'(x) = -2x + 6x^2 - 4x^3 + C_1.$$

We find C_1 by setting $x = 0$ and substituting $f'(0) = 12$:

$$f'(0) = -2(0) + 6(0^2) - 4(0^3) + C_1 \quad \Rightarrow \quad 12 = C_1.$$

Therefore, $f'(x) = -2x + 6x^2 - 4x^3 + 12$. The antiderivative of this is

$$f(x) = -x^2 + 2x^3 - x^4 + 12x + C_2.$$

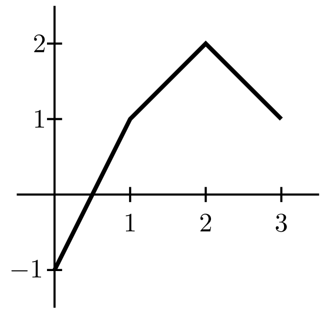
We find C_2 by setting $x = 0$ and substituting $f(0) = 4$:

$$f(0) = -0^2 + 2(0^3) - 0^4 + 12(0) + C_2 \quad \Rightarrow \quad 4 = C_2.$$

Therefore, $f(x) = -x^2 + 2x^3 - x^4 + 12x + 4$.

3.9.47. *The graph of* f' *is shown in the figure. Sketch the graph of* f *if* f *is continuous and* $f(0) = -1$.

Solution. From $x = 0$ to $x = 1$, the graph of f' is at constant $y = 2$; so the graph of f is a line with constant slope 2. From $x = 1$ to $x = 2$, the graph of f' is at constant $y = 1$; so the graph of f is a line with constant slope 1. From $x = 2$ to $x = 3$, the graph of f' is at constant $y = -1$; so the graph of f is a line with constant slope -1. These three line segments must go end-to-end because f is continuous, and the graph must begin at $(0, -1)$ because $f(0) = -1$. Thus the graph of f is as follows:



4.2.33. *The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.*

(a) $\int_0^2 f(x)\,dx$

Solution. This is a trapezoid, and its area is $\frac{1}{2}(1+3)\cdot 2 = 4$.

(b) $\int_0^5 f(x)\,dx$

Solution. We can calculate it in three pieces:

$$\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx.$$

The first piece was done in (a), and we got 4. The second piece is a rectangle with area $3 \cdot 1 = 3$. The third piece is a triangle with area $\frac{1}{2} \cdot 3 \cdot 2 = 3$. Therefore, the total integral is $4 + 3 + 3 = 10$.

(c) $\int_5^7 f(x)\,dx$

Solution. This is a triangle, but it is under the x axis and thus has negative area: $\frac{1}{2} \cdot (-3) \cdot 2 = -3$.

(d) $\int_0^9 f(x)\,dx$

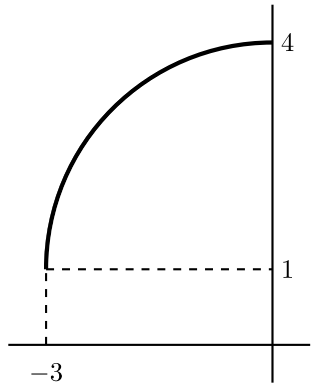
Solution. We can calculate it in three pieces:

$$\int_0^9 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx + \int_7^9 f(x)\,dx.$$

The first two pieces were done in (b) and (c), and we got 10 and -3 respectively. The third piece is a trapezoid, but it is under the x axis and thus has negative area: $-\frac{1}{2}(3+2)\cdot 2 = -5$. Therefore, the total integral is $10 - 3 - 5 = 2$.

4.2.37. *Evaluate the integral* $\int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)dx$ *by interpreting it in terms of areas.*

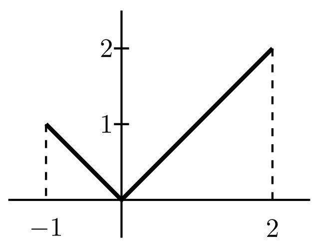
Solution. This is the graph:



The region under the graph is a quarter of a circle (radius 3), stacked on top of a 3×1 rectangle. The integral is the combined area: $\frac{1}{4}\pi \cdot 3^2 + 3 \cdot 1 = \frac{9}{4}\pi + 3 \approx 10.07$.

4.2.39. *Evaluate the integral* $\int_{-1}^{2} |x|\,dx$ *by interpreting it in terms of areas.*

Solution. This is the graph:



The region under the graph is two triangles. The first triangle has area $\frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$, and the second triangle has area $\frac{1}{2} \cdot 2 \cdot 2 = 2$. The integral is the total area of $\frac{1}{2} + 2 = \frac{5}{2} = 2.5$.