October 14, 2008 Names:

Lab #3

Good Times With Cosets

Consider the group $\mathbb{Z}_{12}$ and its subgroup $H = \{0, 4, 8\}$, and define the set

$$\mathcal{C} = \{a + H \mid a \in \mathbb{Z}_{12}\}$$

to be the set of distinct cosets of H in $\mathbb{Z}_{12}$.

1. According to Lagrange's Theorem, how many elements will the set $\mathcal{C}$ have?

2. What is the set $\mathcal{C}$?

3. Suppose we define an operation on $\mathcal{C}$ by

$$(a + H) \star (b + H) = (a + b) + H$$

where $(a + b)$ is addition in $\mathbb{Z}_{12}$. Make a Cayley table of $\mathcal{C}$ under the operation $\star$.

4. Prove that $\mathcal{C}$ is a group under $\star$.

5. What are some interesting properties of $(\mathcal{C}, \star)$? Is it Abelian? Cyclic? etc.

6. Construct a Cayley table of the group $\mathbb{Z}_{12}$ under addition modulo 12, but order the elements of $\mathbb{Z}_{12}$ like so: 0, 4, 8, 1, 5, 9, 2, 6, 10, 3, 7, 11.

7. What do you notice about this Cayley table and the table for $\mathcal{C}$?

8. $(\mathcal{C}, \star)$ should be isomorphic to one of your favorite groups. What is that group? Construct a map from $\mathcal{C}$ to that group and show it is an isomorphism.