

October 21, 2008

Names:

Lab #4

Your New Favorite Crazy Group

Let $G = \{\pm 1, \pm i, \pm j, \pm k\}$, where the operation on the elements obeys the following rules:

$$i^2 = j^2 = k^2 = ijk = -1$$

1. The above equations should tell you how to multiply any two elements of G . Construct the Cayley table for G under this multiplication.

2. Show that G is a group under this multiplication.

3. Consider the subgroup $H = \{1, -1\}$ in G . Is $H \triangleleft G$?

4. What is G/H ? Construct the Cayley table for G/H . Is it a group?

5. What familiar group is G/H isomorphic to?

6. What does this tell us about G ? What “familiar” group is it **not** isomorphic to?

7. Is G isomorphic to any group we have seen before?