Instructions: You are encouraged to work out solutions to these problems in groups! Discuss the problems with your classmates and/or your instructor. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper (this part should be done on your own) and write down the names of the classmates with whom you worked. Be sure to use *complete sentences*. Note: Proofs should contain *words*, not just symbols.

1. (Chapter 9, Exercise 46) If G is a group and $|G : Z(G)| = 4$, prove that $G/Z(G) \cong Z_2 \oplus Z_2$. (You may use - without proof - the fact that any group of order 4 is isomorphic to either Z_4 or $Z_2 \oplus Z_2$.)

2. (Chapter 9, Exercise 54) Show that the intersection of two normal subgroups of G is a normal subgroups of G. Generalize.

3. (Chapter 10, Exercise 32) Find a homomorphism ϕ from $U(30)$ to $U(30)$ with kernel $\{1, 11\}$ and $\phi(7) = 7$.

4. (Chapter 10, Exercise 38) For each pair of positive integers m and n, we can define a homomorphism from $\mathbb{Z}$ to $Z_m \oplus Z_n$ by $x \mapsto (x \ (mod\ m), x \ (mod\ n))$. What is the kernel when $(m, n) = (3, 4)$? What is the kernel when $(m, n) = (6, 4)$? Generalize.