NAME:______________________________

MATH 31 MIDTERM

October 30, 2009

INSTRUCTIONS: For this exam you may use the assigned course text book, your class notes, and any material on the course Blackboard site. If you are confused about what a question is asking of you, you may consult with your instructor. You may not consult any other source.

This exam is due by 5:00 pm on Tuesday, November 3. It should be returned to Paige's office, 221 Kemeny Hall. No credit will be given for late exams.

Staple this page to the front of your completed exam before turning it in. Also, bring this sheet with you any time you discuss this exam with your instructor.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

____

Signature

Question	Points	Score
1	15	
2	15	
3	15	
4	40	
5	30	
6	10	
Total:	125	

Instructions: For this exam you may use the assigned course text book, your class notes, and any material on the course Blackboard site. If you are confused about what a question is asking of you, I encourage you to come see me. I will be happy to clarify the questions for you. If you have trouble getting started, I will "sell" small hints for 2 points each (only one hint per problem). You may not consult any other source.

You may cite a result without proof if it appears in either the assigned text, the assigned homework or your class notes and you should specify exactly where it came from (e.g. Problem 3 of HW2, or Thm. 9.3). Even if you cannot solve one part of a problem, you may still use that result in a later part of the problem. **Important:** Show your work and be sure to explain each answer clearly and completely.

1. [15 points] Classify all abelian groups of order 600, up to isomorphism.

2. (a) [10 points] Use the fact that any subgroup H of a group G having index 2, (i.e. $|G:H|=2$), is normal to find an example of a group G with subgroups K and N such that $K \trianglelefteq N$, $N \trianglelefteq G$, but $K \ntrianglelefteq G$.

   (b) [5 points] You proved in homework that the intersection of two normal subgroups is normal. With G, N and K as in your example, N and K are both normal subgroups, and $K \cap N = K$. Explain why your example doesn't contradict that statement you proved.

3. [15 points] Determine all the homomorphic images of the group $G = Z_9 \oplus Z_5$.

4. This problem considers a group G and its commutator subgroup G'.

   (a) [15 points] Let G be a group and N a normal subgroup of G. Show that the following statements are equivalent:

      1. G/N is abelian.
      2. $xyx^{-1}y^{-1} \in N$ for all $x, y \in G$.

   (b) [10 points] For a group G, let G' be the set of elements of the form $a_1a_2\cdots a_k$, where each a_i is of the form $xyx^{-1}y^{-1}$ for some $x, y \in G$ and k is a positive integer. Prove that this set forms a subgroup of G.

   (c) [10 points] Prove that G' is a normal subgroup of G.

   (d) [5 points] Show that the factor group G/G' is abelian.

5. Let G be an abelian group and let n be a positive integer. Let $G_n = \{g \in G \mid g^n = e\}$ and $G^n = \{g^n \mid g \in G\}$.

   (a) [10 points] Prove that $G_n \leq G$ and explain why this means $G_n \trianglelefteq G$.

   (b) [15 points] Let $\phi : G \to G^n$ by $\phi(g) = g^n$. Show that ϕ is a surjective homomorphism and compute $\ker \phi$.

   (c) [5 points] Prove that $G/G_n \cong G^n$.

6. [10 points] Suppose that $G = G_1 \oplus G_2 \oplus G_3$. Prove that $Z(G) = Z(G_1) \oplus Z(G_2) \oplus Z(G_3)$ (where Z denotes the center of the group).