Jessica Duncan
Melanie Daulton
Robyn Deakins
Jenn Boyle

\# 22. Proof:

For $n=1$, $\frac{n(n+1)}{2} = \frac{1\cdot 2}{2} = 1$

Therefore, this is true.

Now, we assume that for $n-1$,

$$1+2+\ldots+(n-1) = \frac{(n-1)(n)}{2}$$

So for n,

$$1+2+\ldots+\overset{(n-1)}{(n+1)}+n = \frac{(n-1)n}{2}+n = \frac{n^2-n+2n}{2} =$$

$$\frac{n^2+n}{2} = \frac{n(n+1)}{2}$$

Therefore, it is true for all $n \geq 1$.

Comments:

- "typo" in line 6
- very clear
- nice job

Linda C, Vivienne Z,
Matt D, Brian D.

#22. For every positive integer n, prove that: $1+2+\cdots+n = \frac{n(n+1)}{2}$

- Assume $n=1$, then $1 = \frac{1(2)}{2} = 1$, which is true
- Assume it is true for $m \in \mathbb{Z}^+$, so $1+2+\cdots+m = \frac{m(m+1)}{2}$

Then we prove it is true for $m+1$,
add $m+1$ to each side:

$$1+2+\cdots+m+m+1 = \frac{m(m+1)}{2} + (m+1)$$

$$= \frac{m(m+1)}{2} + \frac{2(m+1)}{2} = \frac{m^2+m+2m+2}{2}$$

$$1+2+\cdots+m+m+1 = \frac{(m+1)(m+2)}{2}$$

This formula is of the same format of $1+2+\cdots+n = \frac{n(n+1)}{2}$,

so for every positive integer n, the formula holds. □

Comments:
- Show formula in step 1.
- Very well done.

Sam R
Mine Chin
Zoe
Tommy

#22

choose $n=1$ then $\frac{(1)(1+1)}{2} = 1$ so base case is true

assume it holds for $1 \le k < n$

$n = k+1$

So

$$1+2+\dots+k = \frac{k(k+1)}{2} \quad \text{(inductive hypothesis)}$$

add $k+1$ to both sides

$$1+2+\dots+2k+1 = \frac{k(k+1)}{2} + k+1$$

$$= \frac{k^2+k}{2} + \frac{2k+2}{2}$$

$$= \frac{k^2+3k+2}{2}$$

$$1+2+\dots+k+k+1 = \frac{(k+1)(k+2)}{2}$$

so it holds for $k+1$

$$1+2+\dots+5+n = \frac{n(n+1)}{2}$$

Comments:

- Show the formula in step 1.
- Nice job.
- Give more explanations of steps/ make things more explicit.

Vivienne Zhao, Linda Cummins, Joey Dang

#10 – Let a and b be integers and let $d = \gcd(a,b)$ if $a = da'$ and $b = db'$, show that $\gcd(a', b') = 1$

– <u>assumptions</u>:
 – $a, b \in \mathbb{Z}$
 – $d = \gcd(a,b)$
 – $a = da'$, $b = db'$

– Let c be the greatest common divisor of a' and b'

– There exists an m and n in $\mathbb{Z}$ such that:
$$mc = a' \quad \text{and} \quad nc = b'$$
(Since c divides a' and b')

– By substitution, $a = dmc$ and $b = dnc$

– By commutativity/assoc. $a = m(dc)$ $b = n(dc)$ *

– So dc divides a and b

– Since d is the gcd of a, any other divisor of a must be less than d, so $dc \leq d$

– for this to be true c must be ≤ 1

– However, c must be an integer, so $c = 1$, and is the gcd of a' and b'. □

Comments:

- show all steps
- consider placing the justification before the statement:
 "Since ..., we can see ..."
 or "By ..., we have ..."

Nick Koltes
Mira
Irina
Bill

#10 Let a and b be integers and let $d = \gcd(a,b)$. If $a = da'$ and $b = db'$, show that $\gcd(a', b') = 1$

Assume that:
a and b are integers
d is the greatest common divisor of a and b
1) Assume $a = da'$ and $b = db'$ (1)

let c be the greatest common divisor of a' and b'
Therefore, (2) there exists m and n contained in the Integers such that,
Since we know c divides a' and c divides b', Then

2) $mc = a'$ and $nc = b'$.

By substitution of equation (1) into equation (2), we get

$a = dmc$ and $b = dnc$

By associativity and commutativity,

$a = m(dc)$ and $b = n(dc)$

Therefore, $(dc) | a$ and $(dc) | b$

Since $dc | a$ and $dc | b$ and $d = \gcd(a,b)$, then $dc \leq d$

Comments:
Presentation!

Jen, Robyn, Melanie

#10 ✓ Let c be the greatest common devisor of a' and b'

There exists integers m and n such that

$mc = a'$

$nc = b'$

since a' and b' are divisible by c

substituting this into the assumptions that

$a = da'$

$b = db'$

gives

$a = dmc$

$b = dnc$

Comments:

- Proof flows well
- carefully done
- Very clear. Good job.

by commotivity,

$a = dmc = mdc$ and

$b = dnc = ndc$

by associativity,

$a = (md)c = m(dc)$

$b = (nd)c = n(bc)$

so, $\frac{a}{dc} = m$

and $\frac{b}{dc} = n$

so, by the definition of divisibility,

dc divides a and dc divides b

because d is the greatest common divisor and dc divides a and b, $dc \le d$

so $c \le 1$

since c is the greatest common divisor of a' and b' $c \ge 1$

Sara M. Le
Matt Dinger
Joe Celli

#19 Show that $\gcd(a, bc) = 1$ if and only if $\gcd(a, b) = 1$ & $\gcd(a, c) = 1$

A) if $\gcd(a, bc) = 1$ then $\gcd(a, b) = 1$ & $\gcd(a, c) = 1$
Assume, $\gcd(a, bc) = 1$
That is, a & bc are relatively prime
By thm. 0.2 which states GCD is a linear combination (pg 5)
$\Rightarrow \exists\ m, n \in \mathbb{Z}$ s.th. $ma + nbc = 1$
And by associativity
(1) $ma + (nb)c = 1$ (2) $ma + (nc)b = 1$
So therefore, ~~m, nb ∈ ℤ s.th. we know~~
by (1) $g(a, c) = 1$ where $m, nb \in \mathbb{Z}$
(2) $g(a, b) = 1$ when $m, nc \in \mathbb{Z}$

B) If $\gcd(a, b) = 1$ & $\gcd(a, c) = 1$ then $\gcd(a, bc) = 1$

Assume that $\gcd(a, b) = 1$ & $\gcd(a, c) = 1$
By thm 0.2 (GCD is a linear combination)
let $m, n, m', n' \in \mathbb{Z}$ s.th.
$am + bn = 1$ & $am' + cn' = 1$
So by multiplication
$(am + bn)(am' + cn') = 1$. And by FOIL
$a^2mm' + amcn' + am'bn + bncn' = 1$
By associativity ~~& com~~ & commutativity
$a(mm' + mcn' + m'bn) + (nn')bc = 1$
& by them 0.2

$\gcd(a, bc) = 1$

Comments: • Good, clear, neatly organized.
• use "and" instead of &
• use words in place of symbols like ⇒ and ∃

Samuel R.
Tammy D
Brian D.

#19 $a, b, c \in \mathbb{Z}$

($\Rightarrow$) We are given that $\gcd(a, bc) = 1$
By thm 0.2 $\exists\ m, n \in \mathbb{Z}$ such that $ma + nbc = 1$.
By associativity, $ma + (nb)c = 1$
By thm. 0.2, $\gcd(a, c) = 1$
By associativity and commutativity of mult,
$$ma + (nc)b = 1$$
Hence by 0.2 $\gcd(a, b) = 1$

($\Leftarrow$) It is assumed that $\gcd(a, c) = \gcd(a, b) = 1$
By thm 0.2, $\exists\ m, n, m', n' \in \mathbb{Z}$ such that
$$am + bn = 1 \text{ and } am' + \cancel{b}^{c}n' = 1$$
Multiply together, expand by FOIL, and
$$a^2mm' + amcn' + am'bn + bncn' = 1$$
By associativity and commutativity,
$$a(amm' + mcn' + m'bn) + (nn')bc = 1$$
$(amm' + mcn' + m'bn)$ and (nn') are linear combinations of integers, they are also integers
Hence by 0.2, $\gcd(a, bc) = 1$.

comments: "Typo" on line 3 of part 2.
well written/easy to read.
could use more words in justifications.

Zoe
Damla
Michael

#19

Assume that
$\Rightarrow$ The greatest common divisor of a and bc is 1. There exists integers m and n such that $ma + nbc = 1$ because the greatest common divisor of a and bc is a linear combination. By associativity, $ma + (nb)c = 1$. This implies that the greatest common divisor of a and c is 1 since this is a linear combo with coefficients m and nb. Using associativity & commutativity to rearrange the order of multiplication, we can write $ma + (nc)b = 1$. This implies that the greatest common divisor of a and b is 1 since this is a linear combo with coefficients m and nc.

$\Leftarrow$ Assume that the greatest common divisor of a and c and a and b are both 1. There exists integers m, n, m', n' such that $am + bn = 1$ and $am' + cn' = 1$ since the greatest common divisor is a linear combination. Multiplying these two equations yields $a^2mm' + amcn' + am'bn + bncn' = 1$. Using associativity and commutitivity, we can write $a(amm' + mcn' + m'bn) + (nn')bc = 1$. Since this is a linear combination with coefficients $amm' + mcn' + m'bn$ and nn', this implies that the greatest common divisor of a and bc is 1.

$\square$

Comments: Use more symbols.
Write equations on separate line to distinguish them from the paragraphs
State objective.

Nick Pelfer
Joey Dang
Khai

30 The fibonacci numbers are 1, 1, 2, 3, 5, 8, ...
In general, defined by $f_1 = 1$, $f_2 = 1$,
$f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. Prove
the n^{th} fibonacci number satisfies $f_n < 2^n$

for $a = 1$ $f_1 = 1 < 2^1$
for $a = 2$ $f_2 = 1 < 2^2$
for $a = 3$ $f_3 = 2 < 2^3$

Assume $f_i < 2^i$ is true for $i < n$

for $a = n$, $f_n = f_{n-1} + f_{n-2}$

assumed that $f_{n-1} < 2^{n-1}$ and $f_{n-2} < 2^{n-2}$

Therefore, $f_n < 2^{n-1} + 2^{n-2}$

$$2^{n-2} \leq 2^{n-1} \quad \forall n > 2$$

$$f_n < 2^{n-1} + 2^{n-2} \leq 2^{n-1} + 2^{n-1} = 2 \cdot 2^{n-1} = 2^n$$

Therefore, $f_n < 2^n$

Comments: Good job.

Chen
Irina
Mira

30

$f_1 = 1$
$f_2 = 1$

$\forall n \geq 3 : f_n = f_{n-1} + f_{n-2}$
Prove f_n satisfies : $f_n < 2^n$.

BASE CASE : $n = 3$. $f_3 = f_2 + f_1 = 2$ | $f_3 \leq 2^3$ *
$2^n = 2^3 = 8$ (True)

Inductive Step :
Assume $\forall n$ we have that $f_n < 2^n$ (1)
We also know that $f_n = f_{n-1} + f_{n-2}$ (2)
Show : $\forall n$ $f_{n+1} < 2^{n+1}$
From (1) and (2) we have : $f_{n-1} + f_{n-2} < 2^n$

$f_{n+1} = f_n + f_{n-1} < 2^n + 2^{n-1} = 2^{n-1}(2+1) = 3 \cdot 2^{n-1}$

So we have $f^{n+1} < 3 \cdot 2^{n-1}$
We want $f^{n+1} < 2^{n+1} = 4 \cdot 2^{n-1}$ | $\Rightarrow f^{n+1} < 2^{n+1}$
$3 \cdot 2^{n-1} < 4 \cdot 2^{n-1}$

**

Comments: Better job. Would be somewhat more clear if conclusions * and ** were written below their justifications.

#30

So we have

~~Assume~~ ~~those~~ $f_n = f_{n-1} + f_{n-2}$

Now Assume $f(n-1)$ is true

$$\Rightarrow f(n-1) = f_{(n-1)-1} + f_{(n-1)-2}$$

$$f_{n-1} = f_{(n-2)} + f_{(n-3)}$$

~~for n≥3~~

$$\boxed{\text{show } f_{(n-1)} < 2^{(n-1)}}$$

$$\parallel$$

$$f(n-1) < 2^n - 1$$

$$\Downarrow$$

$$f_{(n-1)} < f_{n-1} + f_{n-2} - 1$$

$$F_n = f_{n-1} + f_{n-2}$$

$$F_4 = F_3 + F_2 < 2^n$$

~~F_4~~ $F_3 < 2^3$, $F_2 < 2^1$

we want to show $(n-1) \geq 3$

$P(n-1)$: $f_{(n-1)} < 2^{(n-1)}$

$f_{(n-1)} < 2^n$

Comments: could be better organized

SARA LE
Sam
Bill Mergin

Base case $n=3$

Show $f_n < 2^n$

whenever $f_k < 2^k$ for $k<n$

$f_n = f_{n-1} + f_{n-2}$

$< 2^{n-1} + 2^{n-2}$

$< 2\,2^{n-1} = 2^n$

since $2^{n-2} < 2^{n-1}$