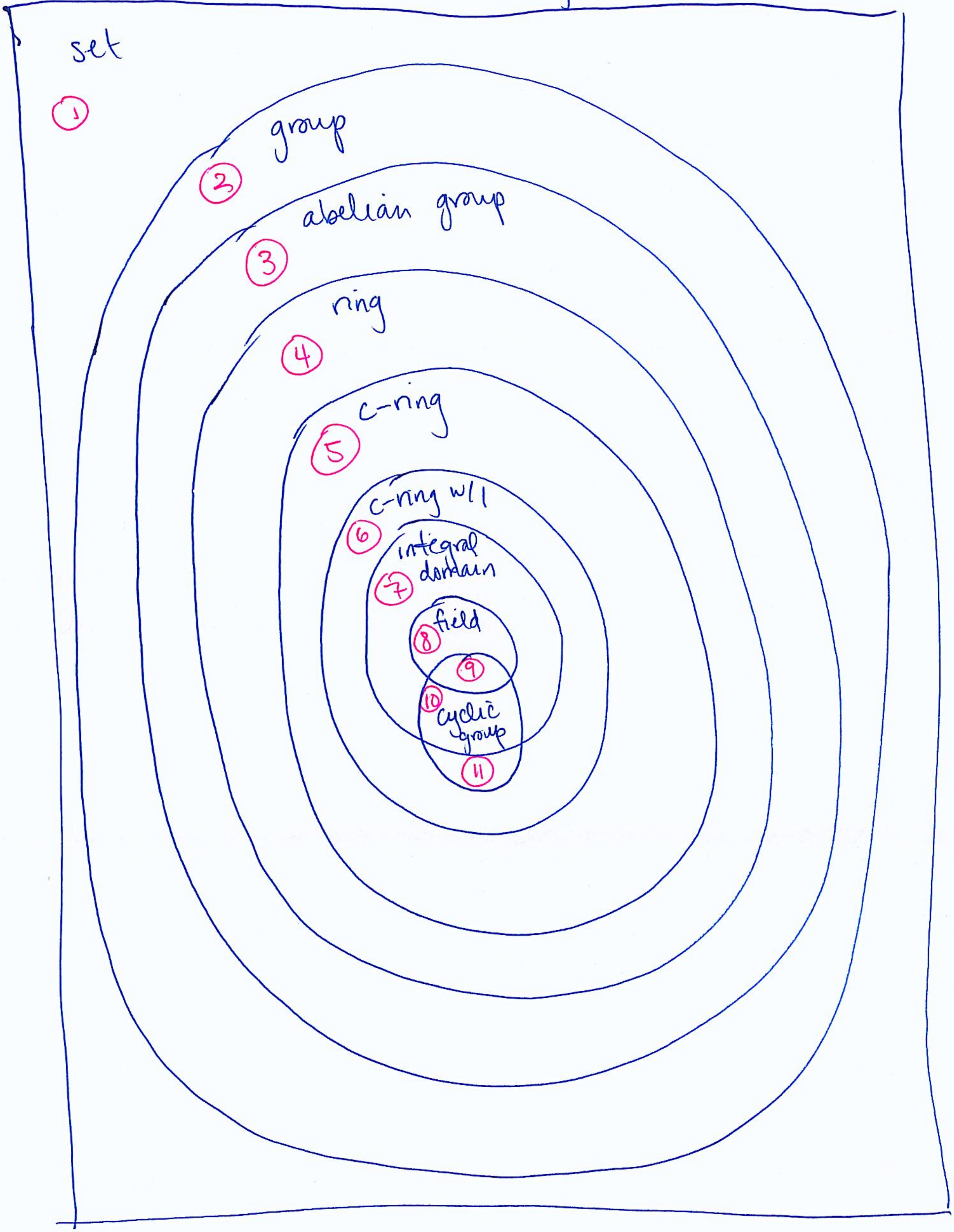
A set A could have the following structures:



Algebraic Structure Examples:

1. The natural numbers, $\mathbb{N}$, form a set but not a group under $+$ or $\cdot$.

2. D_4 is a non-abelian group.

3. The rotations of a square form an abelian group but have no secondary operation and so they do not form a ring.

4. The matrix ring $M_n(\mathbb{R})$ is not commutative

5. The ring of even integers, $2\mathbb{Z}$, has no unity element but is commutative.

6. The ring $\mathbb{Z}_6[x]$ is a c-ring w/1 which is not an integral domain nor a cyclic group.

7. The ring $\mathbb{Z}_7[x]$ is an integral domain which is not a field nor a cyclic group.

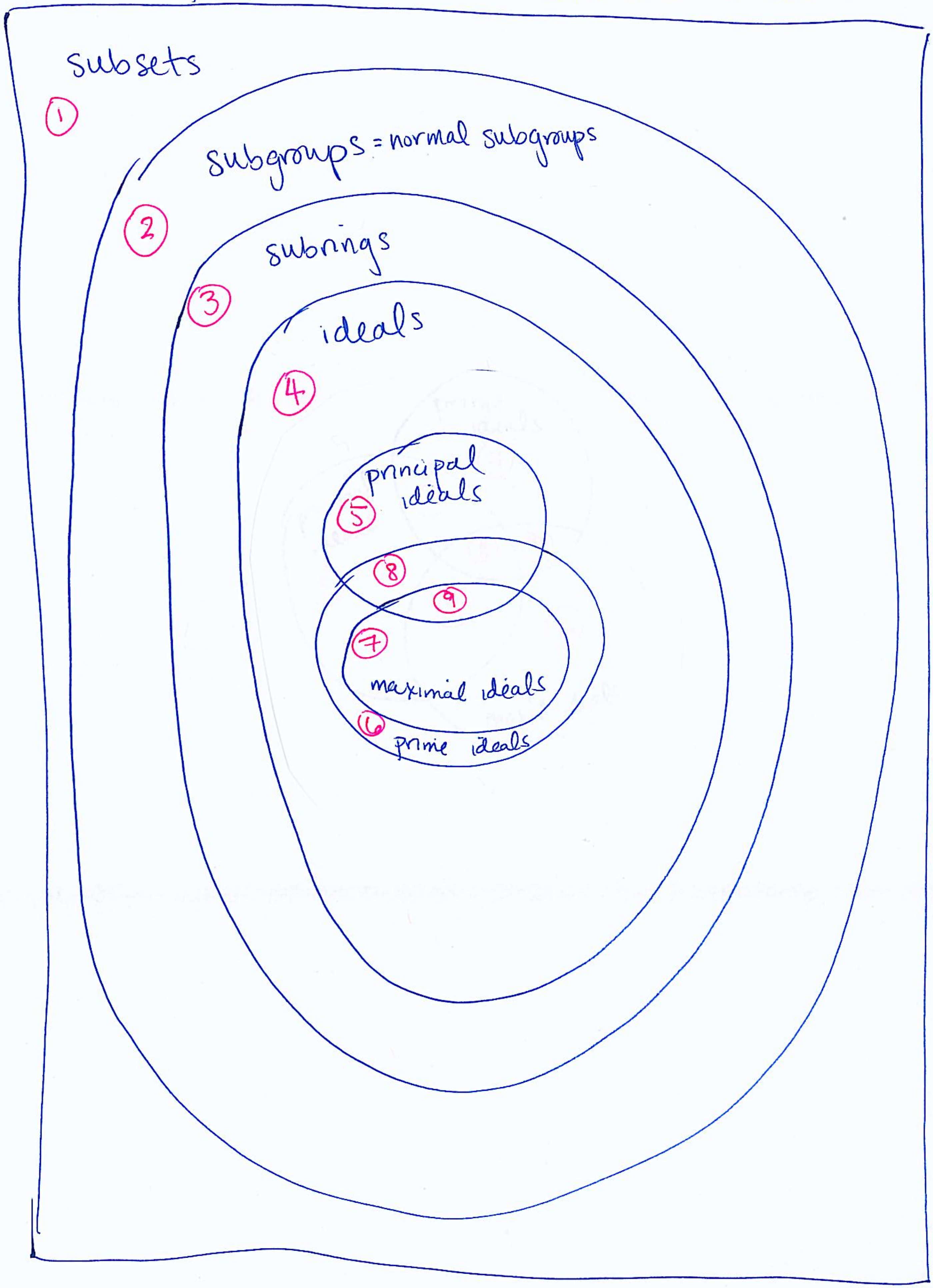
8. $\mathbb{Q}$ is a field which is not a cyclic group.

9. $\mathbb{Z}_7$ is a cyclic group which is a field.

10. $\mathbb{Z}$ is a cyclic group which is an integral domain but not a field.

11. $\mathbb{Z}_6$ is a cyclic group which is a c-ring w/1 but not an integral domain.

For a c-ring w/1 R, a subset $S \subseteq R$ falls into one of:



Algebraic Substructures Examples:
(all inside some c-ring w/1)

1. The subset $\mathbb{N} \subseteq \mathbb{Z}$ is not a subgroup.

2. $\{ f(x) \in \mathbb{Z}[x] \mid \deg(f(x)) = 1 \text{ or } f(x) = 0 \}$ is a subgroup but not a subring of $\mathbb{Z}[x]$.

3. The diagonal 2×2 <u>real</u> matrices form a c-ring w/1. The matrices $\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid a \in \mathbb{Z} \}$ form a subring, but not an ideal.

4. The ideal $\langle 6, x \rangle \subseteq \mathbb{Z}[x]$ is not prime, principal or maximal.

5. The principal ideal $\langle 6 \rangle \subseteq \mathbb{Z}$ is not prime (or maximal).

6. The ideal $\langle x, y \rangle \subseteq \mathbb{Z}[x,y]$ is prime but not maximal nor principal.

7. The ideal $\langle 2, x \rangle \subseteq \mathbb{Z}[x]$ is maximal not principal.

8. The ideal $\langle x \rangle \subseteq \mathbb{Z}[x]$ is prime and principal, but not maximal.

9. The ideal $\langle x^2+1 \rangle \subseteq \mathbb{R}[x]$ is maximal (and prime) and principal.