

Math 31: Abstract Algebra
Fall 2016 - Homework 1

Return date: Wednesday 09/21/16

keywords: *operations - examples and properties*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

The following exercises are from Chapter 2 of the textbook. You will find example solutions to similar problems there.

exercise 1. (*6 points*) Which of the following rules are operations on the indicated set? Justify your answer in each case.

- a) $a * b = \sqrt{|a \cdot b|}$, on the set \mathbb{Q} of rational numbers.
- b) $a * b = a \cdot \ln(b)$, on the set $\{x \in \mathbb{R} : x > 0\}$.
- c) $a * b = |a - b|$, on the set $\{n \in \mathbb{Z} : n \geq 0\}$.

exercise 2. (*6 points*) Each of the following is an operation $*$ on \mathbb{R} .

- a) $x * y = |x + y|$.
- b) $x * y = x \cdot y + 1$.
- c) $x * y = \max\{x, y\}$.

Explain whether or not

- i) the operation is commutative,
- ii) the operation is associative,
- iii) \mathbb{R} has an identity element with respect to $*$,
- iv) every $x \in \mathbb{R}$ has an inverse with respect to $*$.

exercise 3. (*8 points*) Let A be the two-element set $A = \{a, b\}$. Write a table of all 16 possible operations on A using the format explained in the book on page 20. Label these operations Op_1 to Op_{16} . Then

- a) among these operations, identify those that are commutative,
 - b) identify the operations that have an identity element,
 - c) identify the operations for which each element has an inverse.
-