Math 31: Exam 2 Practice

Date: 10/24/19

Test your knowledge

True/false questions

1. The family of sets $\{A_r \mid r \in \mathbb{R}\}$ defined by $A_r = \{(x, y) \mid x^2 + y^2 = r^2\}$ forms a partition of $\mathbb{R} \times \mathbb{R}$. ◯ True ◯ False

2. Let $\langle G, \cdot \rangle$ be a group and H a subgroup. If $a \notin Hb$, then $Ha \neq Hb$. ◯ True ◯ False

3. Let $\langle G, \cdot \rangle$ be a group and H a subgroup. For a fixed $a \in G$ the function $f : aH \to Ha$ defined by $f(ah) = ha$ is a bijective function. ◯ True ◯ False

4. Let $\langle G, \cdot \rangle$ be a group and H a subgroup. Then for any $a \in G$ we have aHa^{-1} is a subgroup of G, where $aHa^{-1} = \{aha^{-1} \mid h \in H\}$. ◯ True ◯ False

5. Let G be a finite group, and let H and K be subgroups of G with $H \subset K$. Then $(G : H) = (G : K)(K : H)$. ◯ True ◯ False

6. Let G be a group. The function $f : G \to G$ defined by $f(x) = x^3$ is a homomorphism. ◯ True ◯ False

7. $\langle \mathcal{F}(\mathbb{R}), +, \cdot \rangle$, where $\cdot$ is function multiplication ($(f \cdot g)(x) := f(x)g(x)$), is a ring. ◯ True ◯ False

8. If $\langle A, +, \cdot \rangle$ is a commutative ring and $b \in A$ a divisor of zero. Then $n \bullet b$ is either zero or a divisor of zero. ◯ True ◯ False

9. If n is not a prime then $\langle \mathbb{Z}_n, +_n, \cdot_n \rangle$ is not an integral domain. ◯ True ◯ False

Note: We have not covered the following material. These are for practice after Friday's class, and solutions will be posted online.

10. If A is a ring and $I \lhd A$ and $J \lhd A$ are ideals then $I \cap J$ is an ideal. ◯ True ◯ False

11. In $\mathbb{Z}_5 \times \mathbb{Z}_5$ the set $B = \{(2n, 2n), n \in \mathbb{Z}_5\}$ is a subring. ◯ True ◯ False

12. In $\mathbb{Z}_5 \times \mathbb{Z}_5$ the set $B = \{(2n, 2n), n \in \mathbb{Z}_5\}$ is an ideal. ◯ True ◯ False

13. Let $\alpha : \langle \mathcal{F}(\mathbb{R}), +, \cdot \rangle \to \langle \mathbb{R}, +, \cdot \rangle$ be the map defined by $\alpha(f) := f(3) - f(0)$. Then α is a ring homomorphism. ◯ True ◯ False

Long answer questions

Question 1 Prove that $m \sim n$ iff $|m| = |n|$ is an equivalence relation on $\mathbb{Z}$. Then, describe the partition associated with that equivalence relation.

Question 2 Write down the cosets of the subgroup $\langle \frac{1}{3} \rangle$ generated by $\frac{1}{3}$.

a) Find all of the cosets of the subgroup $\langle \frac{1}{3} \rangle \leq \langle \mathbb{R}^*, \cdot \rangle$, where $\mathbb{R}^*$ is the set of nonzero real numbers. What is the index of $\langle \frac{1}{3} \rangle$ in $\langle \mathbb{R}^*, \cdot \rangle$?

b) Find all of the cosets of the subgroup $\langle \frac{1}{3} \rangle \leq \langle \mathbb{R}, + \rangle$. What is the index of $\langle \frac{1}{3} \rangle$ in $\langle \mathbb{R}, + \rangle$?

Question 3 Find all the normal subgroups (a) of S_3 and (b) of D_4.

Question 4 Let $K \lhd G$ and $H \lhd K$. Show that if G/H is an abelian group, then both G/K and K/H are abelian groups.