Math 31: Final Exam Practice

Date: 11/18/19

Test your knowledge

True/false questions

1. If A is a ring with n elements and $B \leq A$ a subring. Then $|B|$ divides n. ◯ True ◯ False

2. If $\langle A, +, \cdot \rangle$ is a commutative ring. Then the cyclic subgroup $\langle x \rangle$ of $\langle A, + \rangle$ is equal to the principal ideal $\langle x \rangle$ generated by x. ◯ True ◯ False

3. There are finitely many irreducible polynomials in $\mathbb{Z}_5[x]$. ◯ True ◯ False

4. If $\langle A, +, \cdot \rangle$ is an integral domain with $char(A) = p$, where p prime. Then A has p elements. ◯ True ◯ False

5. The principal ideal $\langle x^2 - 1 \rangle$ in $\mathbb{Z}[x]$ is a prime ideal. ◯ True ◯ False

6. Let A be a ring and J be an ideal of A. Every element of A/J is its own negative if and only if $x + x \in J$ for every $x \in A$. ◯ True ◯ False

7. Every prime ideal of a commutative ring with unity is also a maximal ideal. ◯ True ◯ False

8. All the nonzero elements in a ring have the same additive order. ◯ True ◯ False

9. In $\mathbb{Z}_3[x]$, $x + 2$ is a factor of $x^m + 2$ for all m. ◯ True ◯ False

10. Let A be an integral domain. If $(x + 1)^2 = x^2 + 1$ in $A[x]$, then A must have characteristic 2. ◯ True ◯ False

Long answer questions

Question 1 Let $A \subseteq B$ where A and B are integral domains. Prove that A has characteristic p if and only if B has characteristic p.

Question 3 Compute the field of quotients for integral domain $\mathbb{Z}_5[x]$.

Question 2 Let A be a commutative ring and suppose that a is an idempotent element of A (meaning that $a^2 = a$).

a) Show that the function $f_a : A \to A$ defined by $f_a(x) = ax$ is a ring homomorphism.

b) Describe the kernel and the range of f_a. Be as precise as possible.

c) What does the Fundamental Homomorphism Theorem for rings say about these objects?

Question 4 Show that $p(x)$ is an irreducible polynomial in $F[x]$ (F is a field), then the principal ideal generated by $p(x)$ is a maximal ideal of $F[x]$.

Question 5 Determine if each of the following is irreducible.

1. $x^2 + x + 1$ in $\mathbb{Z}_2[x]$.
2. $x^3 + x + 1$ in $\mathbb{Z}_3[x]$.
3. $x^4 + 1$ in $\mathbb{Z}_{11}[x]$.
4. $x^4 + 10x^2 + 5$ in $\mathbb{Z}[x]$.