Final Exam Study Guide

The final exam is on **Friday, November 22 3pm-6pm** in **Location: Kemeny 105**. The final exam is worth 30% of the final grade. The exam will be cumulative, but new material will account for about 50% of the points.

My office hours for the final week are listed below. Also, feel free to email me at samantha.g.allen@dartmouth.edu or message me via Canvas.

- Tuesday (11/19) 1:30-3:30pm,
- Thursday (11/21) 2-3pm.

Below is a list of topics that will be covered on the final exam. Please refer to the Syllabus on the webpage for "Suggested Practice" problems for getting started with the definitions. I have listed a few additional (and often more challenging) practice problems with each chapter below.

- Chapter 2: Operations, associativity, identity elements, and inverses
  Practice: A.5,6, B.5,6

- Chapter 3: Definition of a group, the groups of integers modulo n, operation tables, definition of Abelian group.
  Practice: B.4,5

- Chapter 4: Cancellation law, uniqueness of identity elements and inverses, the order of a group.
  Practice: B.5,6, C.4,5, D.7

- Chapter 5: Definition of subgroup, the group $\mathcal{F}(\mathbb{R})$ of functions $f : \mathbb{R} \to \mathbb{R}$, subgroups generated by elements, Cayley diagrams.
  Practice: B.4,6, D.3,4, F, G

- Chapter 6: Domian and image of a function, injectivity, surjectivity, bijection, composition of functions, function inverses.
  Practice: C.3,5, G.4

- Chapter 7: Definition of a permutation, permutation groups S_A, symmetric groups S_n, dihedral groups.
  Practice: C.1,2, E.1-4, H.2

- Chapter 8: Cycles and cycle notation, disjoint cycles commute, transpositions, decomposition into transpositions, even and odd cycles.
  Practice: C.4, E, H.1-3

- Chapter 9: Isomorphisms of groups, Cayley's Theorem.
  Practice: D.1-4, E.2,5, G.4, H.3,5

- Chapter 10: Order of group elements
  Practice: H

- Chapter 11: Definition of a cyclic group, Theorem on isomorphisms of cyclic groups, subgroups of cyclic groups, orders of elements in cyclic groups, and orders of cyclic subgroups.
  Practice: A.4,5, B.1,2, D.1,2

- Chapter 12: Definitions of partitions and equivalence classes, the partition determined by equivalence classes, equivalence classes determined by a partition.
  Practice: A.6, B.7,9, D.4, E.1-4

- Chapter 13: Cosets of subgroups of groups, cosets form a partition, Lagrange's Theorem, the index of a subgroup.
  Practice: B.7, D.4, E.4,6, F

- Chapter 14: Homomorphisms of groups, homomorphic images, properties of homomorphisms, definition of normal subgroup, definition of the kernel, the kernel is a normal subgroup of the domain, the range is a normal subgroup of the target.
  Practice: B.4,6, C.5, D.6, E.4, G

- Chapter 15: Definition of a quotient group, facts about cosets of normal subgroups, quotient groups are homomorphic images.
  Practice: C.4,7, D.2, F

- Chapter 16: The Fundamental Homomorphism Theorem, the First Isomorphism Theorem (Exercise F).
  Practice: B, I

- Chapter 17: Definitions of rings, integral domains, fields, the cancellation property, zero divisors, and various other related terms.
  Practice: B, E, F

- Chapter 18: Definitions of subrings, ideals, and ring homomorphisms. The kernel is an ideal, the range is a subring.
  Practice: B.5, C.6,7,9, D.4,5, E.5, F.5, G.4, H.1-3

- Chapter 19: Cosets of ideals of rings, properties of cosets, quotient rings, the Fundamental Homomorphism Theorem for rings, prime ideals, maximal ideals. Theorems about quotients by prime/maximal ideals resulting in integral domains/fields.
  Practice: D.3, E.6

- Chapter 20: Additive order of elements in rings, the characteristic of a ring with unity, theorems about characteristic in integral domains. Every finite integral domain is a field. Fields of quotients.
  Practice: A.4, B.3, F.1-3

- Chapter 21-23: Not covered.

- Chapter 24: Rings of polynomials. Degrees of polynomials, sums, and products. A ring of polynomials over an integral domain is also an integral domain. The division algorithm for polynomials over a field.
  Practice: A.7, B.4,5,6, D.1-2, E

- Chapter 25: Every ideal of $F[x]$ is principal, if F is a field. Definitions of mutliples, associates, gcd, and facts about these and related objects. Euclid's lemma for polynomials, unique factorization thm.
  Practice: D.5, E.1

- Chapter 26: Roots of polynomials over fields, solving equations over fields, Fermat's Little Theorem (without proof). Factoring polynomials over $\mathbb{Z}$ and $\mathbb{Q}$. The rational roots theorem, Eistenstein's Irreducibility Criterion.
  Practice: F. 3, G, I.1-3