Homework 8

(Factoring polynomials, solving polynomial equations, Eisenstein's irreducibility criterion.)

Due Monday, November 18 at 2:10pm in class.

Note: Be sure to justify your answers. No credit will be given for answers without work/justification. In addition, all written homework assignments should be neat and well-organized; **Part A and Part B should be submitted separately**.

Part A:

(1) (a) For each $c \in \mathbb{Z}_7$, determine if $x^2 + c$ is reducible or irreducible over $\mathbb{Z}_7$. Justify your answer.

(b) Let p be prime. How many polynomials of the form $x^2 + c$ for $c \in \mathbb{Z}_p$ are reducible over $\mathbb{Z}_p$? How many are irreducible over $\mathbb{Z}_p$? Explain your reasoning.

(2) (a) Use Fermat's Little Theorem to find a polynomial of degree less than 11 which determines the same function as $a(x) = 7x^{42} + 1$ in $\mathbb{Z}_{11}[x]$.

(b) Factor the resulting polynomial in part (a) so that it is of the form

$$kp_1(x)p_2(x)\cdots p_n(x)$$

where $k \in \mathbb{Z}_{11}$ and each $p_i(x)$ is a monic irreducible polynomial in $\mathbb{Z}_{11}[x]$.

(c) Solve the equation $7x^{42} + 1 = 0$ over $\mathbb{Z}_{11}$.

Part B:

(1) Let $a(x) \in \mathbb{Q}[x]$ be given by

$$a(x) = \frac{1}{2} + \frac{5}{12}x - \frac{1}{4}x^2 - \frac{1}{3}x^3.$$

(a) Find a polynomial $b(x)$ with integer coefficients which has the same roots as $a(x)$.

(b) Apply Theorem 4 from Chapter 26 of the text to your result from part (a) to find a list of possible roots of $b(x)$.

(2) Chapter 26 D.1-2