Homework Assignment 1

Due Wednesday April 4

1. Show that the (short) distance from a point to a straight line in $\mathbb{R}^2$ is the length of the perpendicular. Use vector notation.

2. Show that $\|(x,y)\| = \sqrt[3]{x^3+y^3}$ is a norm on $\mathbb{R}^2$, Does it comes from inner product?

3. Derive the following identities

   - $$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$
   - $$\|\mathbf{u}+\mathbf{v}\|^2 - \|\mathbf{u}-\mathbf{v}\|^2 = 4(\mathbf{u},\mathbf{v})$$

4. If $(\mathbf{u},\mathbf{v}) = 0$ and $(\mathbf{v},\mathbf{w}) = 0$ does that imply that $(\mathbf{u},\mathbf{w}) = 0$?

5. - Does the choice $\|\mathbf{u}\| = \max_{1\le i\le n} |u_i|$, for $\mathbb{R}^n$ is a norm?
   - Does the choice $\|\mathbf{u}\| = \min_{1\le i\le n} |u_i|$, for $\mathbb{R}^n$ is a norm?