Homework Assignment 2

Due Wednesday April 11

1. Expand each vector $\mathbf{u}$ in terms of the orthogonal basis $e_1 = (2, 1, 3)$, $e_2 = (1, -2, 0)$, $e_3 = (6, 3, -5)$.

   - $\mathbf{u} = (9, -2, 4)$
   - $\mathbf{u} = (0, 1, 5)$
   - $\mathbf{u} = (0, 5, 0)$
   - $\mathbf{u} = (1, 0, 0)$
   - $\mathbf{u} = (3, -1, 1)$
   - $\mathbf{u} = (1, 2, 3)$

2. Expand each of the $\mathbf{u}$ vectors in Exercise 1 in terms of the orthonormal basis$\hat{e_1}, \hat{e_2}, \hat{e_3}$ of R^3, where $\hat{e_1}, \hat{e_2}, \hat{e_3}$ are normalized versions of e_1, e_2, e_3 given in Exercise 1.

3. In each case use the Gram-Schmidt process to obtain an orthonormal set from the given linearly independent set.

   (a) $(4, 0)$, $(2, 1)$

   (b) $(1, -2)$, $(3, 4)$

   (c) $(1, 1, 0)$, $(2, -1, 1)$, $(1, 0, 3)$

   (d) $(1, 1, 1)$, $(2, 0, -1)$

   (e) $(1, 1, 1)$, $(1, 0, 1)$, $(1, 1, 0)$

4. In $\mathbb{R}^5$ find the best approximation to the given $\mathbf{u}$ vector within $span(e_1, e_2, e_3)$,

   $$e_1 = \frac{1}{\sqrt{5}}(1, 0, 2, 0, 0), \qquad e_2 = \frac{1}{\sqrt{6}}(2, 0, -1, 0, 1), \qquad e_3 = (0, 0, 0, 1, 0)$$

   and the norm of the error vector.

   - $(3, -2, 0, 0, 5)$
   - $(3, 0, 1, 4, 1)$
   - $(0, 2, 0, 0, 0)$

5. In $\mathbb{R}^4$, let

$$e_1 = \frac{1}{\sqrt{3}}(1,1,0,-1), \quad e_2 = \frac{1}{\sqrt{3}}(1,-1,-1,0),$$
$$e_3 = \frac{1}{\sqrt{3}}(1,0,1,1), \quad e_4 = \frac{1}{\sqrt{3}}(0,1,-1,1)$$

Find the best approximation to $\mathbf{u} = (4,-2,1,6)$ within $span\{e_1\}$, $span\{e_1, e_2\}$, $span\{e_1, e_2, e_3\}$, and $span\{e_1, e_2, e_3, e_4\}$ and in each case compute the norm of the error vector.

6. Prove the Bessel inequality in $\mathbb{R}^n$. Let $\{e_1, e_2, \ldots, e_k\}$ be orthogonormal set prove:

$$\sum_{i=1}^{k}(\mathbf{u}, e_i) \leq \|\mathbf{u}\|^2$$