

Homework Assignment 5

Due Friday May 4

1. Finding Green's function for the heat equation:

- (a) use the equation $\int_0^\infty e^{-(a^2x^2+b^2x^{-2})}dx = \frac{\sqrt{\pi}}{2a}e^{-2ab}$, $a > 0$, $b \geq 0$. to show that:

$$\mathcal{F}(e^{-t^2}) = \frac{1}{\sqrt{2}}e^{-\frac{s^2}{4}}$$

Hint: $t^2 - ist = (t - \frac{1}{2}is)^2 + \frac{1}{4}s^2$.

- (b) if $\mathcal{F}\{f(t); s\} = F(s)$ Show that.

$$\mathcal{F}\{f(at); s\} = \frac{1}{a}F\left(\frac{s}{a}\right) \quad a > 0$$

- (c)

$$\mathcal{F}\{e^{-a^2t^2}; s\} = \frac{1}{\sqrt{2}a}e^{-\frac{s^2}{4a^2}} \quad a > 0$$

- (d)

$$\mathcal{F}\{e^{-\frac{t^2}{2}}; s\} = e^{-\frac{s^2}{2}}$$

- (e) Show that:

$$\mathcal{F}^{-1}\{e^{-a^2s^2t}; x\} = \frac{1}{a\sqrt{2t}}e^{-\frac{x^2}{4a^2t}}$$

2. From the book page 95 ex 3