Definition 1 *Vector space is a non empty set* $\mathbf{V}$ *if the following are met:*

1. *An operation, which will be called vector addition and denoted as* $+$*, is defined between any two vectors in* $\mathbf{V}$ *in such a way that if* $\mathbf{u}$ *and* $\mathbf{v}$ *are in* $\mathbf{V}$*, then* $\mathbf{u}+\mathbf{v}$ *is too (i.e.,* $\mathbf{V}$ *is closed under addition). Furthermore,*

$$\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u} \qquad \textit{(commutative)}$$

$$(\mathbf{u}+\mathbf{u})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w}). \qquad \textit{(associative)}$$

2. $\mathbf{V}$ *contains a unique zero vector* $\mathbf{0}$ *such that*

$$\mathbf{u}+\mathbf{0}=\mathbf{u}$$

for each $\mathbf{u}$ *in* $\mathbf{V}$*.*

3. *For each* $\mathbf{u}$ *in* $\mathbf{V}$ *there is a unique vector "*$-\mathbf{u}$*" in* $\mathbf{V}$*, called the negative inverse of u, such that*

$$\mathbf{u}+(-\mathbf{u})=0$$

.

4. *Another operation, called scalar multiplication, is defined such that if* $\mathbf{u}$ *is any vector in* $\mathbf{V}$ *and* α *is any scalar, then the scalar multiple* $\alpha\mathbf{u}$ *is in* $\mathbf{V}$*, too (i.e.,* $\mathbf{V}$ *is closed under scalar multiplication). Further, we require that*

$$\alpha(\beta\mathbf{u})=(\alpha\beta)\mathbf{u} \qquad \textit{(associative)}$$
$$(\alpha+\beta)\mathbf{u}=\alpha\mathbf{u}+\beta\mathbf{u} \qquad \textit{(distributive)}$$
$$\alpha(\mathbf{u}+\mathbf{v})=\alpha\mathbf{u}+\alpha\mathbf{v} \qquad \textit{(distributive)}$$
$$l\mathbf{u}=\mathbf{u}$$

,

if the vectors $\mathbf{u},\mathbf{v}$ *are in* $\mathbf{V}$*, and* α,β *are scalars.*

Definition 2 *vector space* $\mathbf{H}$ *is called an inner product space if to each pair of vectors* $\mathbf{u}$ *and* $\mathbf{v}$ *in* $\mathbf{H}$ *is associated a number* $(\mathbf{u}, \mathbf{v})$ *such that the following rules hold:*

1. $$(\mathbf{u}, \mathbf{v}) = (\mathbf{v}, u)$$

2. $$(\mathbf{u} + \mathbf{w}, \mathbf{v}) = (\mathbf{u}, \mathbf{v}) + (\mathbf{w}, \mathbf{v})$$

3. $$(\alpha\mathbf{u}, \mathbf{v}) = \alpha(\mathbf{u}, \mathbf{v})$$

4. $$(\mathbf{u}, \mathbf{u}) \geq 0$$

5. $$(\mathbf{u}, \mathbf{u}) = 0 \Longleftrightarrow \mathbf{u} = 0$$

Definition 3 *vector space* $\mathbf{V}$ *is called an norm space if to each vector* $\mathbf{u}$ *in* $\mathbf{V}$ *is associated a number* $\|\mathbf{u}\|$ *such that the following rules hold:*

1. $$\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|v\| \qquad \text{(triagle inequality)}$$

2. $$\|\alpha\mathbf{u}\| = |\alpha|\|\mathbf{u}\|$$

3. $$\|\mathbf{u}\| = 0 \Longleftrightarrow \mathbf{u} = 0$$