Review for Finale

1. From the book Page 288 Ex. 2,3,4

2. Determine the values of a, b, c, d and e that minimize the integral:

$$\int_{-1}^{1} (x^5 - ax^4 - bx^3 - cx^2 - dx - e)^2 dx$$

3. Find the complex form of the Fourier series of the following functions:

   (a)
$$f(x) = cosh(ax) \quad -\pi < x < \pi$$

   (b)
$$f(x) = \cos(ax) \quad -\pi < x < \pi$$

   (c)
$$f(x) = \cos(2x) + 3\cos(3x) \quad -\pi < x < \pi$$

4. Use D'Alembert's method to solve:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad 0 \le x \le 1 \qquad 0 < t$$

$$u(0,t) = 0 \qquad u(1,t) = 0$$

$$u(x,0) = f(x) \qquad \frac{\partial u}{\partial}(x,0) = g(x)$$

   (a)
$$f(x) = \sin(\pi x) + 3\sin(2\pi x), \qquad g(x) = \sin(\pi x),$$

   (b)
$$f(x) = 0, \qquad g(x) = -10,$$

5. solve:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial u}{\partial t} \qquad -\pi \le x \le \pi \qquad 0 < t$$

$$u(-\pi,t) = u(\pi,t), \qquad u_x(-\pi,t) = u_x(\pi,t)$$
$$u(x,0) = |x|$$

6. solve:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} + u \qquad 0 \le x \le \pi \qquad 0 < t$$

$$u_x(0,t) = 0, \qquad u_x(\pi,t) = 0$$
$$u(x,0) = x^2$$

7. Find the Fourier transform of:

   (a)
   $$f(x) = \frac{\sin(ax)}{x}$$

   (b)
   $$f(x) = \frac{a - ix}{a^2 + x^2}$$

8. Solve:

$$t\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \qquad -\infty < x < \infty \qquad 0 < t$$
$$u(x,0) = f(x)$$

9. Find the Laplace transform of:

   (a)
   $$\sqrt{t} + \frac{1}{\sqrt{t}}$$

   (b)
   $$te^{-t}\sin(t)$$

10. Find the inverse Laplace transform of:

$$\frac{2s-1}{s^2 - s - 2}$$

11. Solve

$$\nabla u(r,\theta) = 0 \qquad 0 < r < \rho, \qquad -\pi < \theta < \pi$$
$$u(\rho,\theta) = \cos^2(\theta)$$