Math 36 — Weekly Homework
Assigned: 10/15
Due: 10/22

1. Should we expect vote totals in voting precincts to follow Benford's Law? Why or why not?

2. Suppose you are on a Monty Hall gameshow with with 100 doors. After you pick a single door, the host opens 70 doors.

   (a) What is the probability that the door you chose has the prize?

   (b) What is the probability that any one of the unpicked, unopened doors has the prize?

   (c) Find the answer to (b) for a more general game with $n$ total doors and $m$ opened doors.

3. In this problem, we will use our new knowledge about Markov chains to analyze the Birth-death process. Suppose we have a population with 5 individuals, one type A with fitness 2 and the other four being type B with fitness 1. We will make this a Markov chain where state $i$ means $i$ individuals of type A. (You are free to use a computer to do the matrix calculations, particularly the matrix inverse.)

   (a) Draw this Markov chain. Include all the transition probabilities.

   (b) Which states are absorbing? Which states are transitive?

   (c) Give the transition matrix $P$ in **canonical form**. Be sure to label the rows and columns so it is clear which state is being represented.

   (d) Give the matrices $Q$, $R$, $N = (I - Q)^{-1}$, and $B = NR$.

   (e) What is the probability that type A takes over the population?

   (f) What is the expected value for the time it will take for A to either fixate or go extinct?