1. Can you decompose the Petersen graph into copies of

   (a) P_3 :  ?

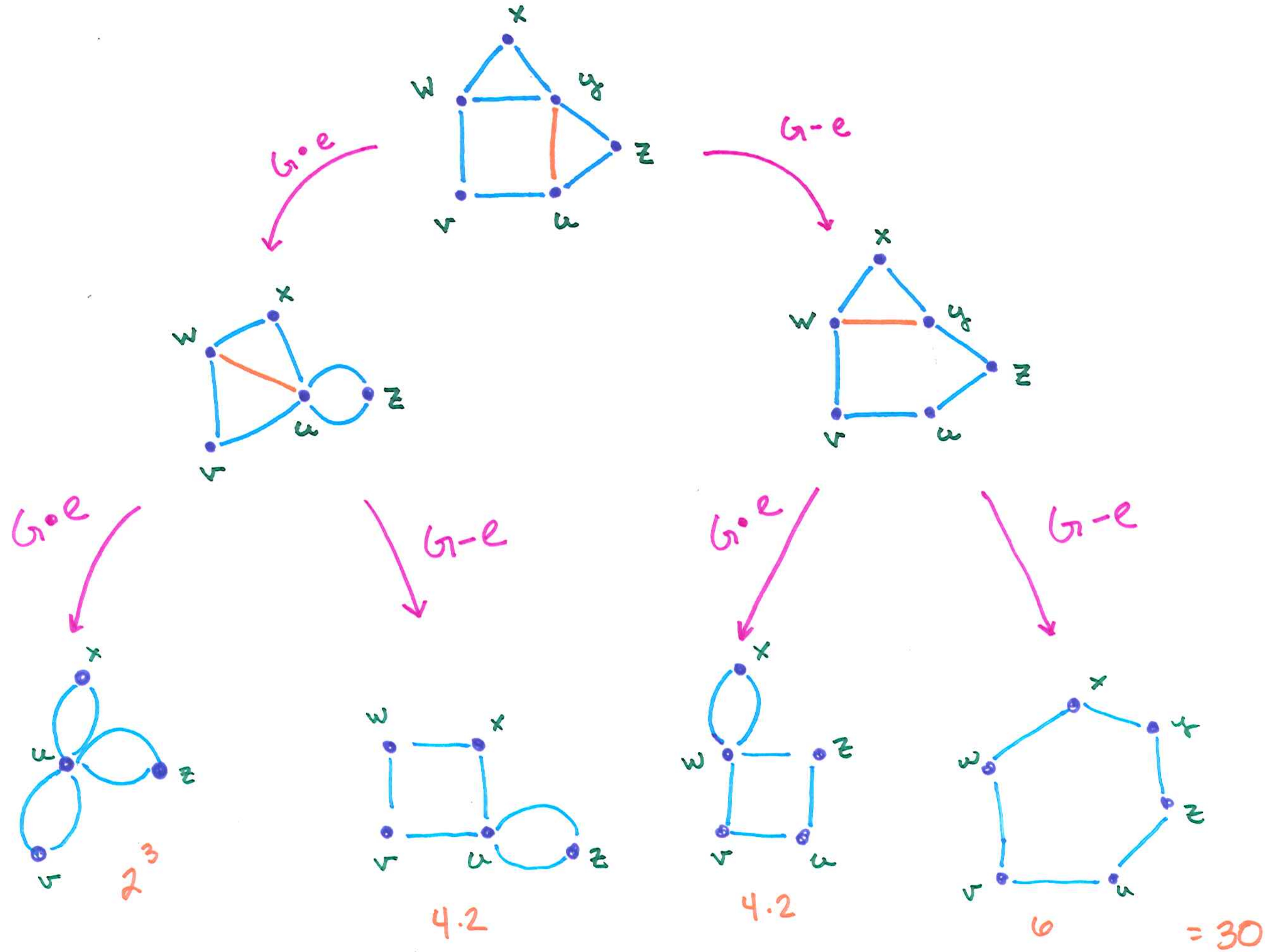
   (b) P_4 :  ?

   (c) $K_{1,3}$:  ?

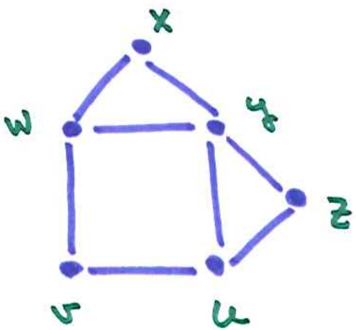
2. Can you decompose K_7 into copies of $K_{1,3}$?

3. Are there any (other) trees you can decompose K_7 into?

   (count edges)



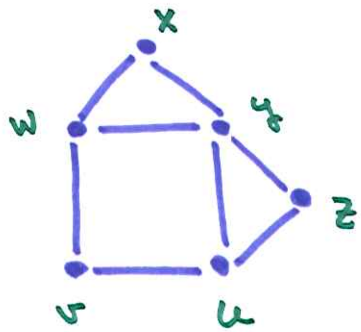
x
w
y
z
v
u
G•e
G-e
x
w
z
u
v
x
w
y
z
v
u
G•e
G-e
G•e
G-e
x
u
z
v
2^3
w
x
v
u
z
4·2
x
w
z
v
u
4·2
x
w
y
z
v
u
6
= 30



$$D = \begin{array}{c} \\ u \\ v \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccccc} u & v & w & x & y & z \end{array} \\ \begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \end{array}$$

$$A = \begin{array}{c} u \\ v \\ w \\ x \\ y \\ z \end{array} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$Q = \begin{array}{c} \begin{array}{cccccc} u & v & w & x & y & z \end{array} \\ \begin{pmatrix} 3 & -1 & 0 & 0 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & -1 & 4 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} \end{array} \begin{array}{c} \\ u \\ v \\ w \\ x \\ y \\ z \end{array}$$



$$D = \begin{array}{c c} & \begin{array}{cccccc} u & v & w & x & y & z \end{array} \\ \begin{array}{c} u \\ v \\ w \\ x \\ y \\ z \end{array} & \begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \end{array}$$

$$Q = \begin{array}{c c} \begin{array}{cccccc} u & v & w & x & y & z \end{array} & \\ \begin{pmatrix} 3 & -1 & 0 & 0 & -1 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & -1 & -1 & 4 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} & \begin{array}{c} u \\ v \\ w \\ x \\ y \\ z \end{array} \end{array}$$

$$A = \begin{array}{c c} \begin{array}{c} u \\ v \\ w \\ x \\ y \\ z \end{array} & \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$Q_{4,3} = \begin{pmatrix} 3 & -1 & 0 & -1 & -1 \\ -1 & 2 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ -1 & 0 & -1 & 4 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}$$

$$Q_{5,5} = \begin{pmatrix} 3 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & 0 \\ -1 & 0 & 0 & 0 & 2 \end{pmatrix}$$