Quiz 2, Math 38, Spring 2012

Instructions:
Always (briefly) explain or demonstrate why, unless told otherwise.
In counting problems, answers like "$\binom{5}{2} + 2^4 + 3 * 5$" are usually preferable to "41".

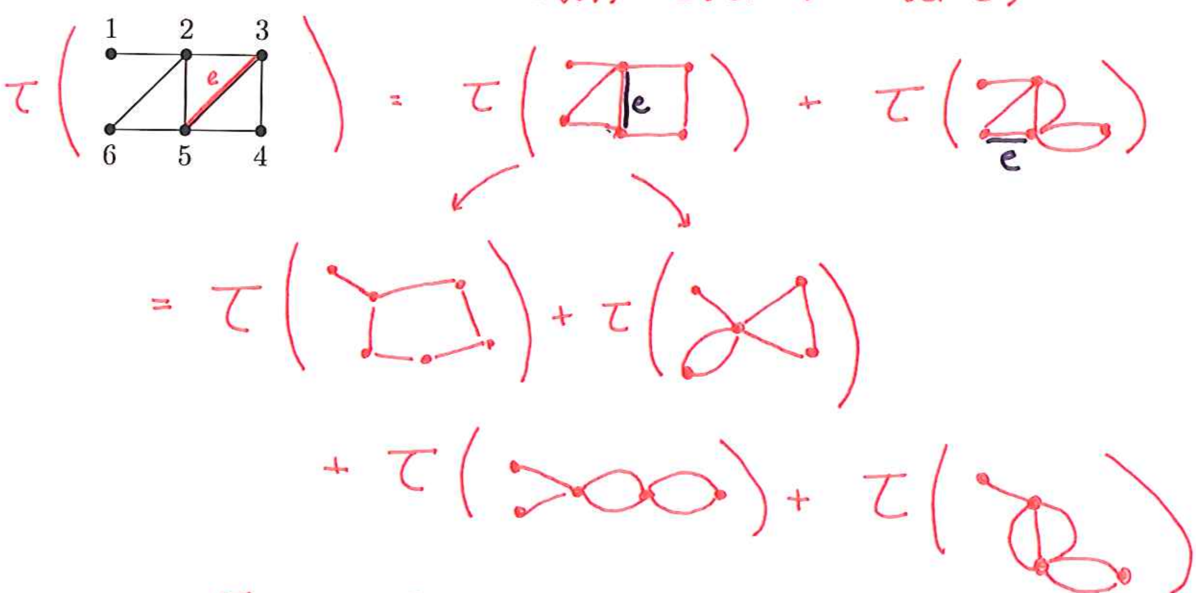
(1) How many spanning trees does K_5 have?

Since K_5 has every tree on 5 vertices,
K_5 has

$n^{n-2} = 5^3$ sp. trees

(2) How many spanning trees does the following graph have?

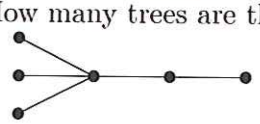
$\tau(G) = \tau(G-e) + \tau(G \cdot e)$



$= 5 + 2*3 + 2*2 + 3*2$

$= \boxed{21}$

(3) How many trees are there in the same isomorphism class as the one below?



$n = 6$

One vertex of degree 4 & one of degree 2
(uniquely identifies those vertices)

Then 3 vertices adjacent to the $d(v)=4$ vertex
that are leaves.

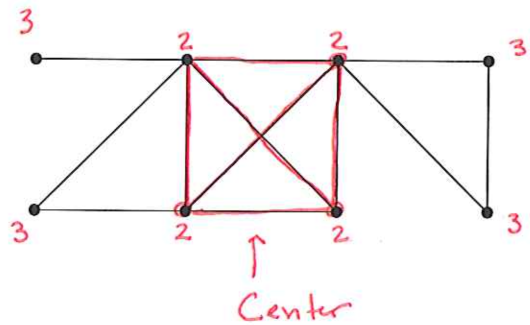
$6 * 5 * \binom{4}{3} = \boxed{120}$

(4) For the following graph,
   (a) label each vertex with its eccentricity,
   (b) trace over the center of G,
   (c) give its diameter,
   (d) give its radius.

No need to justify.

diam = 3

rad = 2



(5) For the same graph as above, without drawing the complement $\bar{G}$, can you give upper and/or lower bounds on the diameter of $\bar{G}$?

Since $\operatorname{diam}(G) \geq 3$,

we have $\operatorname{diam}(\bar{G}) \leq 3$.

Also G has edges, so

$\operatorname{diam}(\bar{G}) \geq 2$.

(6) Is the following statement true or false? If true, explain why. If false, give a counter-example and a similar statement which is true. (no need to prove)

A graph G is a tree if ~~or~~ and only if every edge is a cut edge.

False: e  is not a tree, but e is a cut edge.

Better

A graph G is a tree if and only if it is connected and every edge is cut.

-or-

A graph G is a forrest iff every edge is cut.

(7) State four (other?) ways to characterize trees with n vertices.
(No need to prove, just state them. Don't reuse (6).)
Some set of properties X characterize trees on n vertices if you can make the statement "A graph G with n vertices is a tree if and only if G has properties X".

(1) G is connected and acyclic

(2) G is connected w/ n-1 edges

(3) G is acyclic w/ n-1 edges

(4) for every pair u, v ∈ E(G), there is exactly one u-v path in G, and G is loopless.

(5) Adding any edge w/ endpts in V(G) creates exactly one cycle, and G is loopless.