

HW 12

2.15 #3

$$\int \left(\frac{x^9}{9} - \frac{x^5}{5} + \frac{x^3}{3} - 1 \right) dx$$

$$= \frac{x^{10}}{90} - \frac{x^6}{30} + \frac{x^4}{12} - x + C$$

note: $\left(\frac{x^{10}}{90} \right)' = \frac{\cancel{10}x^9}{\cancel{90}9} = \frac{x^9}{9}$

Likewise for other terms.

#5 $\frac{dy}{dx} = x^{\frac{1}{9}}$

$$y = \int x^{\frac{1}{9}} dx = \frac{x^{\frac{10}{9}}}{\frac{10}{9}} + C$$

$$= \frac{9}{10} x^{\frac{10}{9}} + C$$

$$y(1) = 8 \Rightarrow \frac{9}{10} \cdot 1 + C = 8 \Rightarrow C = 8 - \frac{9}{10} = 7.1$$

$$y = \frac{9}{10} x^{\frac{10}{9}} + 7\frac{1}{10}$$

~~#~~ $-\infty < x < \infty$

#6

$$y = \int \sin(8x) dx = \frac{-\cos(8x)}{8} + C$$

$$y\left(\frac{\pi}{8}\right) = 1 \Rightarrow \frac{-\cos\pi}{8} + C = 1 \Rightarrow C = \frac{7}{8} \quad \left(\frac{-\cos\pi}{8} = \frac{-(-1)}{8} = \frac{1}{8} \right)$$

$$y = \frac{-\cos(8x)}{8} + \frac{7}{8}$$

$$-\infty < x < \infty$$

#7

$$y' = \int (7x + \sin(x) + 10) dx$$

$$\Downarrow$$

$$y' = \frac{7}{2}x^2 - \cos x + 10x + C$$

$$y'(0) = 0 \Rightarrow -1 + C = 0 \Rightarrow C = 1$$

$$\Downarrow$$

$$\begin{aligned} y &= \int \left(\frac{7}{2}x^2 - \cos x + 10x + 1 \right) dx \\ &= \frac{7}{6}x^3 - \sin x + 5x^2 + x + C \end{aligned}$$

$$y(0) = 14 \Rightarrow C = -14$$

$$\Downarrow$$

$$y(x) = \frac{7}{6}x^3 - \sin x + 5x^2 + x - 14$$

$$-\infty < x < \infty$$

#8

$$y' = \int (3x^2 - 5) dx$$

$$= x^3 - 5x + C$$

$$y'(0) = 5 \Rightarrow C = 5$$

⇓

$$y = \int (x^3 - 5x + 5) dx$$

$$= \frac{x^4}{4} - \frac{5}{2}x^2 + 5x + C$$

$$y(0) = 0 \Rightarrow C = 0$$

⇓

$$y = \frac{x^4}{4} - \frac{5}{2}x^2 + 5x$$

~~0 < x < 100~~