Hour Exam #2

Math 3

Oct. 31, 2012

Name (Print): ____
Last First

On this, the second of the two Math 3 hour-long exams in Fall 2012, and on the final examination I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:________________________________

Instructor (circle):

Lahr (Sec. 1, 8:45) Diesel (Sec. 2, 10:00)
Dorais (Sec. 3, 11:15) Dorais (Sec. 4, 12:30)
Wolff (Sec. 5, 1:45)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on page 1 of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 12 multiple-choice problems worth 5 points each and 3 long-answer written problems worth a total of 40 points. Check to see that you have 10 pages of questions plus the cover page for a total of 11 pages.

Non-multiple choice questions:

Problem	Points	Score
13	10	
14	15	
15	15	
Total	40	

1. The number of students attending a Dartmouth event increases at a rate proportional to the number of students present. If the only people at the event initially are the organizing committee (5 students), but after 90 minutes there are 250 students present, how many students were there 45 minutes into the event?

   (a) $5e^{\frac{\ln(50)}{2}}$
   (b) $e^{\frac{\ln(50)}{90}}$
   (c) 175
   (d) $245e^{\frac{\ln(50)}{5}}$
   (e) None of the above

S = # students present at time t

$\frac{dS}{dt} = kS$, $S(0) = 5$ $S(90) = 250$ $S(45) = ?$

$S = Ce^{kt}$

$5 = Ce^0 = C$

$250 = S(90) = 5e^{90k}$

$50 = e^{90k}$

$\frac{\ln(50)}{90} = k$

$S(45) = 5e^{\frac{\ln(50) \cdot 45}{90}} = 5e^{\frac{\ln(50)}{2}}$

2. While at the event, you get a cold soda. This soda will warm at a rate proportional to the difference between its own temperature and that of the surrounding environment. The room is 80 °F and the soda is initially 40 °F, but after 10 minutes is 50 °F. If you model the situation by $T = T_E + Ce^{kt}$, where T_E is the environment temperature and C and k are constants, what is k?

   (a) $k = \frac{\ln(3/4)}{10}$
   (b) $k = \ln(3/4)$
   (c) $k = -40$
   (d) $k = 40\ln(-3/4)$
   (e) None of the above

$T = 80 + Ce^{kt}$

$40 = T(0) = 80 + Ce^0$

$-40 = C$

$T = 80 - 40e^{kt}$

$50 = T(10) = 80 - 40e^{k(10)}$

$\frac{-30}{-40} = e^{10k} \Rightarrow \frac{\ln(\frac{3}{4})}{10} = k$

3. If a ball is thrown upward with an initial velocity of 19.6 m/s, how long does it take to reach its maximum height?

   (Recall that the downward acceleration due to gravity is $9.8\,\mathrm{m/s^2}$.)

   (a) 1 second
   (b) 2 seconds
   (c) 4 seconds
   (d) 8 seconds
   (e) None of the above

$\frac{dv}{dt} = -9.8$

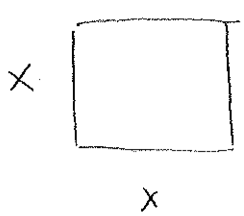
$v = -9.8t + v(0)$

$0 = -9.8t + 19.6$

$2 = \frac{-19.6}{-9.8} = t$

4. The area of a square is expanding at constant rate of $3\,\mathrm{m^2/s}$. How fast is the perimeter of the square expanding when the square has side length 6 m?

   (a) 0.5 m/s
   (b) 1.0 m/s
   (c) 1.5 m/s
   (d) 2.0 m/s
   (e) None of the above



$\frac{dA}{dt} = 3$

$A = x^2$

$\sqrt{A} = x$

$P = 4x$

$P = 4\sqrt{A}$

$x = 6$

$A = 6^2 = 36$

$\frac{dP}{dt} = \frac{1}{2} \cdot 4 (A)^{-1/2} \frac{dA}{dt}$

$\frac{dP}{dt} = \frac{2}{\sqrt{A}} \frac{dA}{dt} = \frac{2}{6} \cdot 3 = 1$

5. Solve the initial value problem (IVP):

$$\frac{dy}{dx} = (1-2x)y; \qquad y(1) = -5$$

(a) $y = \ln|1-2x| - 5$
(b) $y = e^{x-x^2}$
(c) $y = x - x^2 - 5$
(d) $y = -5e^x$
(e) None of the above

$\int \frac{1}{y} dy = \int (1-2x) dx$

$\ln|y| = x - x^2 + C$

$y = Ce^{x-x^2}$

$-5 = y(1) = Ce^0 = C$

$y = -5e^{x-x^2}$

6. Solve the initial value problem (IVP):

$$y\frac{dy}{dx} - \sin(x) = 1; \qquad y(0) = 3$$

(a) $y = x - \cos(x) + 4$
(b) $y = \sqrt{2x - 2\cos(x) + 11}$
(c) $y = \sqrt{4x - 4\cos(x) + 7}$
(d) $y = \sqrt{x - \cos(x) + 3}$
(e) None of the above

$y\frac{dy}{dx} = 1 + \sin x$

$y\,dy = (1 + \sin x)\,dx$

$\frac{1}{2}y^2 = x - \cos x + C$

$y^2 = 2x - 2\cos x + C$

$y = \pm\sqrt{2x - 2\cos x + C}$

$3 = y(0) = \sqrt{-2 + C}$

$9 = -2 + C \Rightarrow C = 11$

$y = \sqrt{2x - 2\cos x + 11}$

7. Estimate a root of $f(x) = x^3 + x + 1$ using two iterations of Newton's Method with starting with $x_0 = 0$.

(a) $x_2 = -1$

(b) $x_2 = -3/4$

(c) $x_2 = 0$

(d) $x_2 = 1/2$

(e) Newton's Method cannot be used for this problem.

$f'(x) = 3x^2+1$

$x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_0 = 0$

$x_1 = -1$

$x_1 = 0 - \frac{1}{1}$

$= -1$

$x_2 = -1 - \frac{f(-1)}{f'(-1)} = -1 - \frac{-1}{4} = -1 + \frac{1}{4} = -\frac{3}{4}$

8. Let $f(x) = (x+6)^{2/3}$. Find the linearization of $f(x)$ at $x = 2$ and use it to approximate $f(1.9)$.

(a) 121/30

(b) 4.1

(c) $\sqrt[3]{62.41}$

(d) 119/30

(e) None of the above

$f(x) = (x+6)^{2/3}$

$f(2) = (8^{1/3})^2 = 2^2 = 4$

$f'(x) = \frac{2}{3}(x+6)^{-1/3}$

$f'(2) = \frac{2}{3}(8)^{-1/3} = \frac{2}{3 \cdot 2} = \frac{1}{3}$

$L(x) = 4 + \frac{1}{3}(x-2)$

$L(1.9) = 4 + \frac{1}{3}(1.9-2)$

$= 4 + \frac{1}{3}\left(-\frac{1}{10}\right)$

$= 4 - \frac{1}{30}$

$= \frac{120-1}{30} = 119/30$

9. Integrate: $\int \left(5x - 3x^2 + \frac{x^5}{3}\right) dx$

(a) $5x^2 - 3x^3 + \frac{1}{3}x^6 + C$

(b) $5 - 6x + \frac{5}{3}x^4 + C$

(c) $5x^2 - 6x^3 + \frac{5}{3}x^6 + C$

(d) $\frac{5}{2}x^2 - x^3 + \frac{1}{18}x^6 + C$

(e) None of the above

$\frac{5}{2}x^2 - x^3 + \frac{x^6}{18} + C$

10. Integrate: $\int \left(e^{-x} + \cos(x)\right) dx$

(a) $-e^{-x} + \sin(x) + C$

(b) $-e^{-x} - \sin(x) + C$

(c) $e^{-x} + \sin(x) + C$

(d) $e^{-x} - \sin(x) + C$

(e) None of the above

$-e^{-x} + \sin(x)$

11. Which of the following is a solution to the differential equation $y' = \frac{x}{y}$?

(a) $y = \ln(x^2+1)$
(b) $y = x^2+1$
(c) $y = \sqrt{x^2+1}$
(d) $y = \sec^2(x)$
(e) None of the above

$y' = \frac{x}{y}$

$\int y\,dy = \int x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$y^2 = x^2 + C$

$y = \pm\sqrt{x^2+C}$

12. Which of the following is a solution of the differential equation $y'' + 9y = 0$?

(a) $y = \sin(3x)$ $\quad y' = 3\cos 3x \quad y'' = -9\sin 3x \quad y'' + 9y = -9\sin 3x + 9\sin 3x = 0$
(b) $y = e^{3x}$
(c) $y = \cos(x/3)$
(d) $y = \ln(3e^x)$
(e) None of the above

13. Use Euler's Method with step size $\Delta x = 1$ to approximate $y(4)$ for the initial value problem (IVP):

$$\frac{dy}{dx} = y + \frac{x}{2}; \qquad y(1) = 1.$$

List all the points and slopes used in the table and include your calculations below.

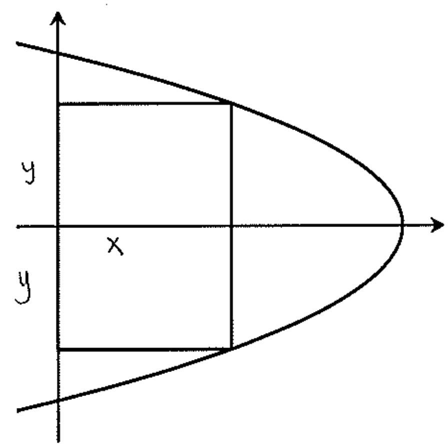
x	y	dy/dx
1	1	$\frac{3}{2}$
2	$\frac{5}{2}$	$\frac{7}{2}$
3	6	$\frac{15}{2}$
4	$\frac{27}{2}$	$\frac{31}{2}$

$$\frac{dy}{dx} = y + \frac{x}{2}$$

point	slope	new x	new y
$(1,1)$	$1+\frac{1}{2}=\frac{3}{2}$	$1+1=2$	$\frac{3}{2}\cdot 1+1=\frac{5}{2}$
$(2,\frac{5}{2})$	$\frac{5}{2}+\frac{2}{2}=\frac{7}{2}$	$2+1=3$	$\frac{7}{2}\cdot 1+\frac{5}{2}=\frac{12}{2}=6$
$(3,6)$	$6+\frac{3}{2}=\frac{15}{2}$	$3+1=4$	$\frac{15}{2}\cdot 1+6=\frac{27}{2}$
$(4,27/2)$	$\frac{27}{2}+\frac{4}{2}=\frac{31}{2}$		

14. Consider a rectangle with left side along the y-axis inscribed in the parabola $x + y^2 = 4$ as illustrated on the right.

What is the largest possible perimeter of such a rectangle? Justify your answer completely.



max: $2x + 4y$

subject to: $x + y^2 = 4$ $\quad x, y \in \mathbb{R}$

$x = 4 - y^2$

max: $f(x) = 2(4-y^2) + 4y = 8 - 2y^2 + 4y$

$f'(x) = -4y + 4$

$0 = -4y + 4$

$y = 1$

+ −

1

max at $1 = y$

$\Rightarrow x = 4 - y^2 = 4 - 1 = 3$

$\Rightarrow P = 2x + 4y$

$= 2(3) + 4 = \boxed{10}$

15. For this problem, show all work and explain your steps in order to receive full credit. Let $f(x) = \dfrac{x^2+9}{x}$.

(a) What are the horizontal and vertical asymptotes of $f(x)$? If there are none, write 'none'.

none

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to 0} f(x) = \pm\infty \Rightarrow$ vertical asymptote at 0

(b) What are the intervals of increase and decrease for $f(x)$?

$$f'(x) = \frac{x(2x) - (x^2+9)}{x^2} = \frac{2x^2 - x^2 - 9}{x^2} = \frac{x^2-9}{x^2} = \frac{(x+3)(x-3)}{x^2}$$

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Increase: $(-\infty, -3) \cup (3, \infty)$

Decrease: $(-3, 0) \cup (0, 3)$

(c) What are the intervals on which $f(x)$ is concave up? Concave down?

$$f''(x) = \frac{x^2(2x) - (x^2-9)(2x)}{x^4} = \frac{2x^3 - 2x^3 + 18x}{x^4} = \frac{18}{x^3}$$

− +

0

concave up: $(0, \infty)$

concave down: $(-\infty, 0)$

(d) Does $f(x)$ have local maximums? Minimums? If so, list them and the function value at these points. If not, write 'none'.

local max at $x = -3$ $\quad f(-3) = \frac{18}{-3} = -6$

local min at $x = 3$ $\quad f(3) = \frac{18}{3} = 6$

Using all the previous information, sketch the graph of $f(x)$: