Trigonometry
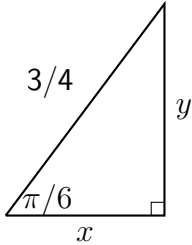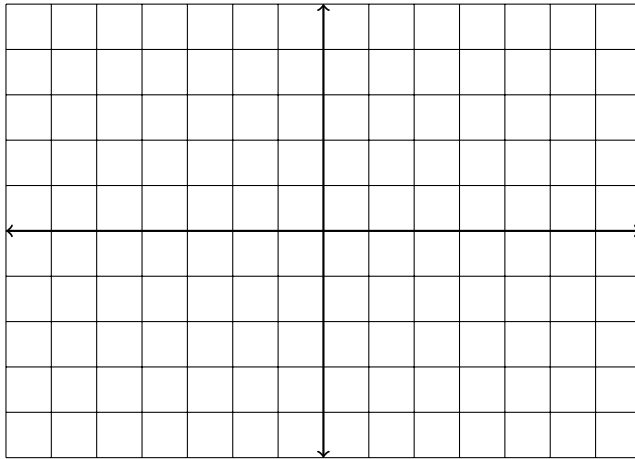
(1) Given the following right triangle, what are the values of x and y?


(2) Determine whether the following trig functions are even or odd:
- $f(x) = \sin x$
- $f(x) = \cos x$
- $f(x) = \tan x$
- $f(x) = \csc x$
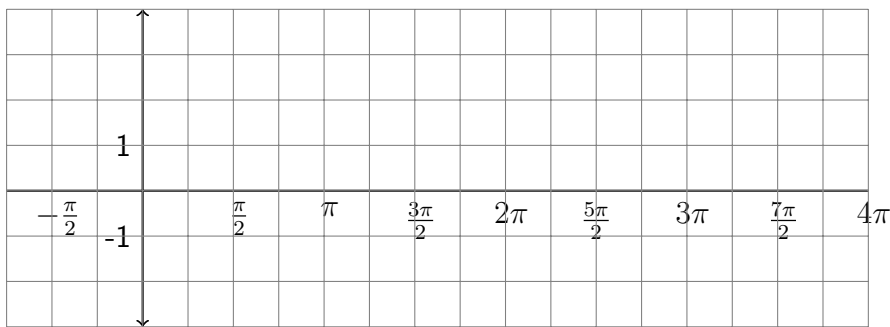- $f(x) = \sec x$
- $f(x) = \cot x$

(3) (a) Consider the domain and range of $\tan x$. For what values of x between 0 and 2π is $\tan x$ undefined? What is the range of $\tan x$? Use this information and the unit circle to help you graph it.
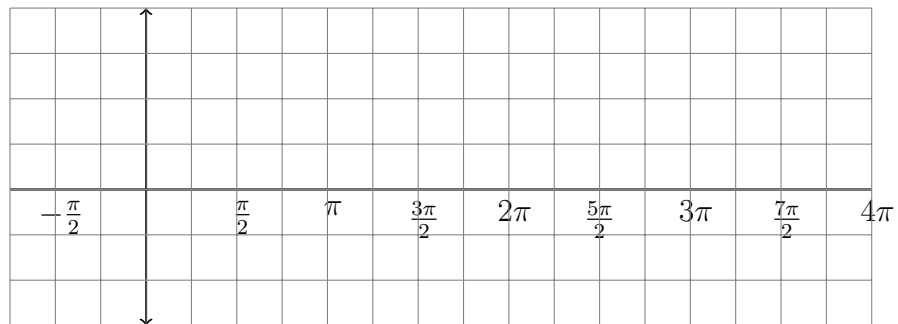
(b) For each of $\cot x$, $\csc x$, $\sec x$, determine what x values between 0 and 2π are not in the domain of the function in question. Additionally, determine the range of each function, and graph it.
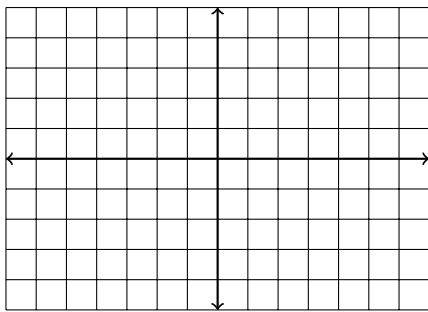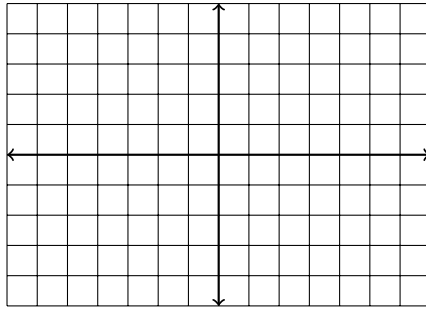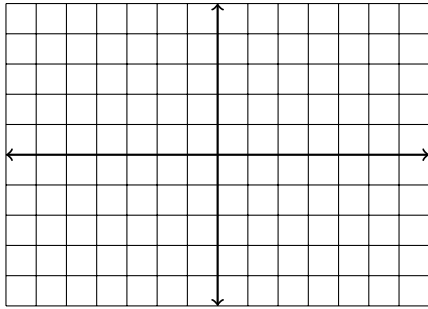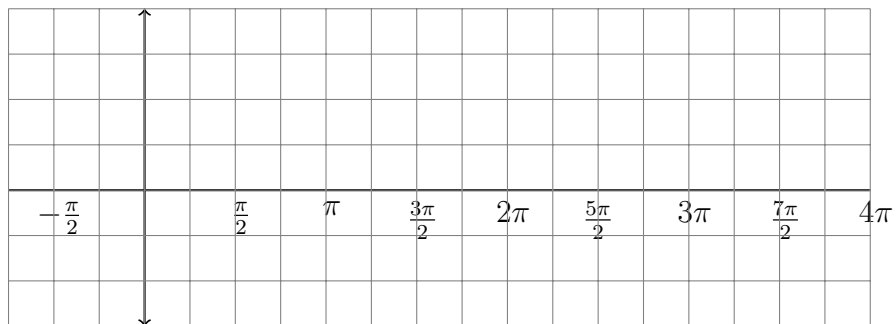
(4) Transform the graph of $f(x) = \cos x$ into the graph of $-2\cos(\frac{1}{2}x - \frac{\pi}{4}) + 1$.
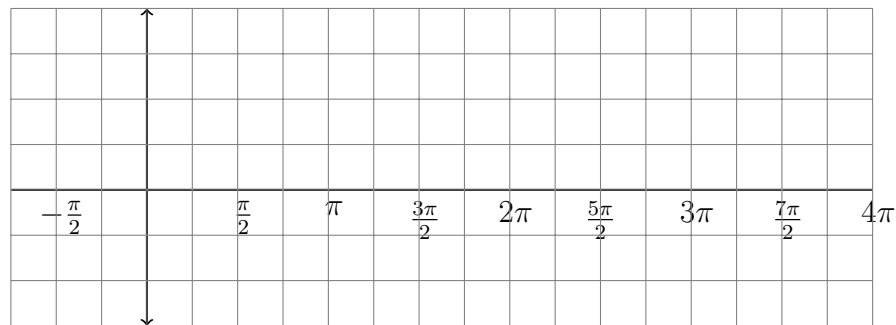First graph $y = \cos(x)$ on the grid below:

(1) $\cos\left(\frac{1}{2}x\right)$

$-\frac{\pi}{2}$ $\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π $\frac{5\pi}{2}$ 3π $\frac{7\pi}{2}$ 4π

(2) $\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

$-\frac{\pi}{2}$ $\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π $\frac{5\pi}{2}$ 3π $\frac{7\pi}{2}$ 4π

(3) $2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

$-\frac{\pi}{2}$ $\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π $\frac{5\pi}{2}$ 3π $\frac{7\pi}{2}$ 4π

(4) $-2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

$-\frac{\pi}{2}$ $\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π $\frac{5\pi}{2}$ 3π $\frac{7\pi}{2}$ 4π

(5) $-2\cos(\frac{1}{2}x - \frac{\pi}{4}) + 1$

| | $-\frac{\pi}{2}$ | | $\frac{\pi}{2}$ | π | $\frac{3\pi}{2}$ | 2π | $\frac{5\pi}{2}$ | 3π | $\frac{7\pi}{2}$ | 4π |
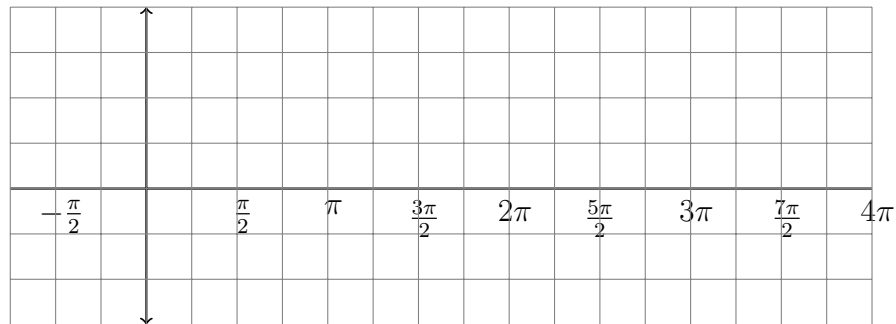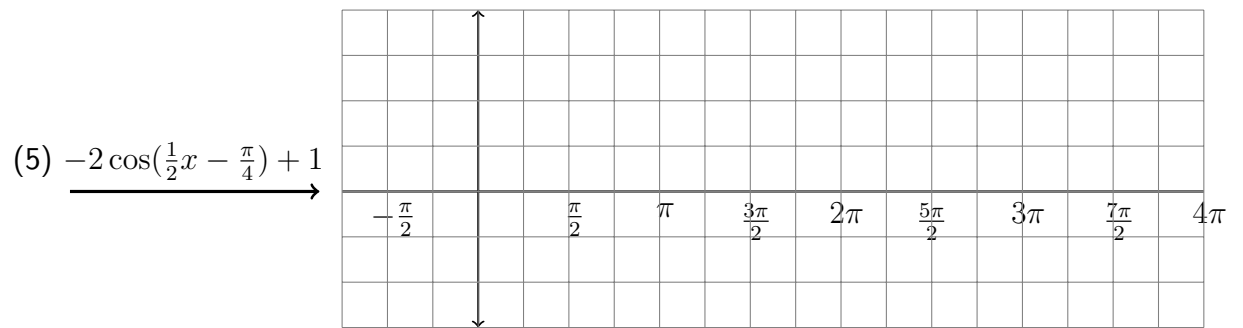
What is the period and amplitude of the result?

(5) Simplify the following expressions using trig identities:
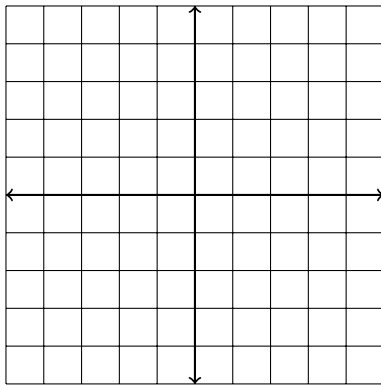
(a) $\cos(2\theta) + 2\sin^2\theta$

(b) $\dfrac{\sin(2x)}{\cos x \sin x}$

(c) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$ (hint: Don't forget to F.O.I.L.!)

Exponents and Logarithms

(1) In class, we drew the graph of $y = a^x$ when $a > 1$. When $a \leq 1$, we get a graph that looks different!
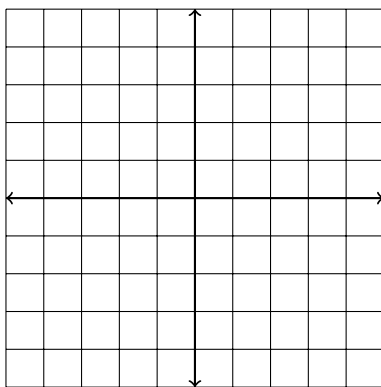
    (a) Warm-up: What happens when $a = 1$? Draw the graph of $y = 1^x$. What is the domain and range of this function?

Domain:

Range:

    (b) Now let $a = \frac{1}{2}$. Draw the graph of $y = \left(\frac{1}{2}\right)^x$. What is the domain and range of this graph? What are the biggest differences between this graph and the graph of $y = a^x$ when $a > 1$?

Domain:

Range:

    (c) Consider the case when $a < 0$. This function would be very difficult to graph. Why do you think that is?

(2) Condense the following logarithmic expressions into something that contains only one log with no coefficients (i.e. $\log(\text{stuff})$).

    (a) $\frac{1}{2}\ln(x) + 3\ln(x+1)$

    (b) $2\ln(x+5) - \ln(x)$

    (c) $\frac{1}{3}(\log_3(x) + \log_3(x+1))$

(3) Solve the following for x:

    (a) $e^{-x^2} = e^{-3x-4}$

    (b) $3(2^x) = 24$

    (c) $2(e^{3x-5}) - 5 = 11$

    (d) $\ln(3x+1) - \ln(5) = \ln(2x)$