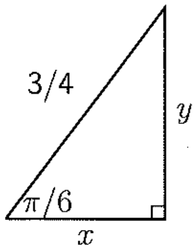
Trigonometry

(1) Given the following right triangle, what are the values of x and y?



$\sin\frac{\pi}{6} = \frac{y}{(3/4)}$ $\qquad$ $\cos\frac{\pi}{6} = \frac{x}{(3/4)}$

$1/2 = 4y/3$ $\qquad$ $\sqrt{3}/2 = 4x/3$

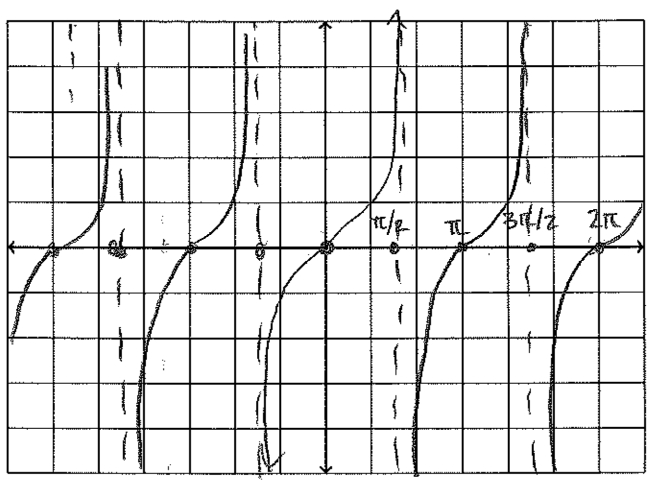
$3 = 8y$ $\qquad$ $3\sqrt{3} = 8x$

$\boxed{\frac{3}{8} = y \qquad \frac{3\sqrt{3}}{8} = x}$

(2) Determine whether the following trig functions are even or odd:

- $f(x) = \sin x$ - odd
- $f(x) = \cos x$ - even
- $f(x) = \tan x$ - odd $\quad \tan x = \frac{\sin x}{\cos x} \Rightarrow \tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$
- $f(x) = \csc x = 1/\sin x$ = odd
- $f(x) = \sec x = 1/\cos x$ even
- $f(x) = \cot x$ $1/\tan x$ odd

(3) (a) Consider the domain and range of $\tan x$. For what values of x between 0 and 2π is $\tan x$ undefined? What is the range of $\tan x$? Use this information and the unit circle to help you graph it.



tan x undefined for $x = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$

Range: $(-\infty, \infty)$

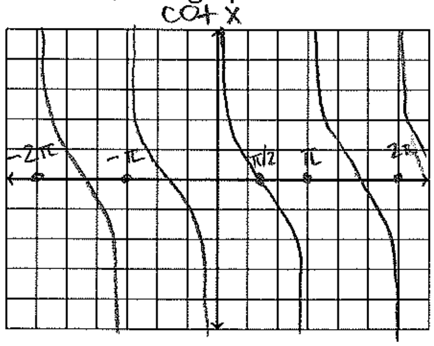
(b) For each of $\cot x$, $\csc x$, $\sec x$, determine what x values between 0 and 2π are not in the domain of the function in question. Additionally, determine the range of each function, and graph it.

$y = \cot x$

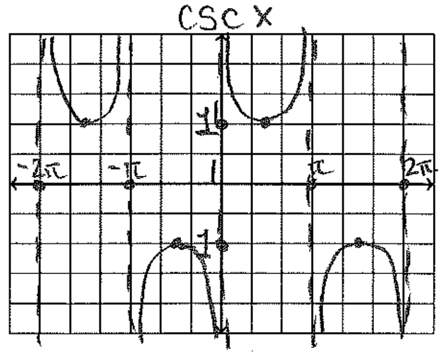
$x \neq 0, \pm\pi, \pm 2\pi, \ldots$

R: $(-\infty, \infty)$

cot x



csc x



$\sin x \neq 0$

$x \neq 0, \pm\pi, \pm 2\pi, \ldots$

Range:

$(-\infty, -1] \cup [1, \infty)$

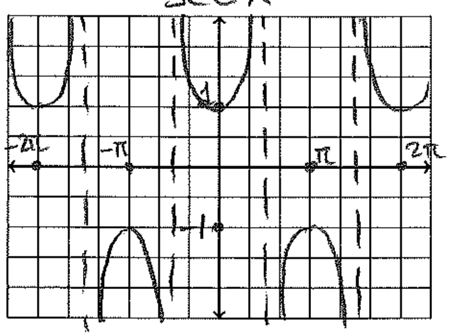
$\cos x \neq 0$

$x \neq \pm\pi/2, \pm 3\pi/2, \ldots$

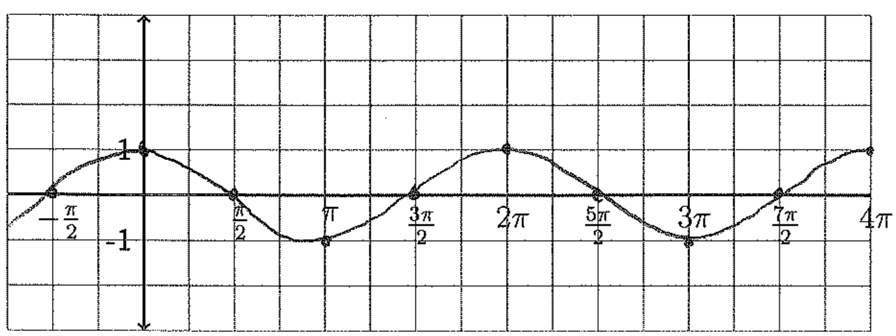
Range:

$(-\infty, -1] \cup [1, \infty)$

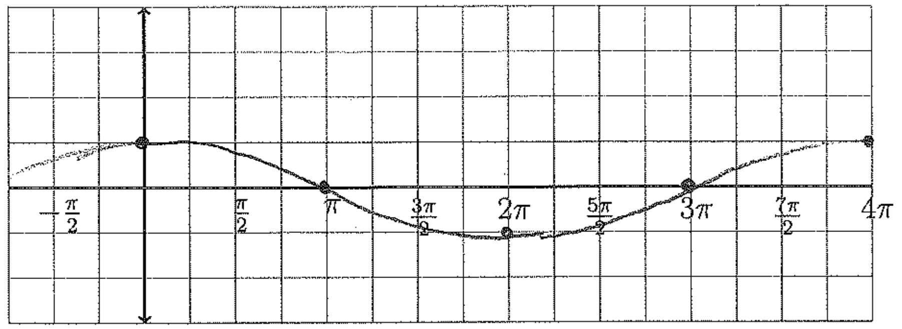
sec x



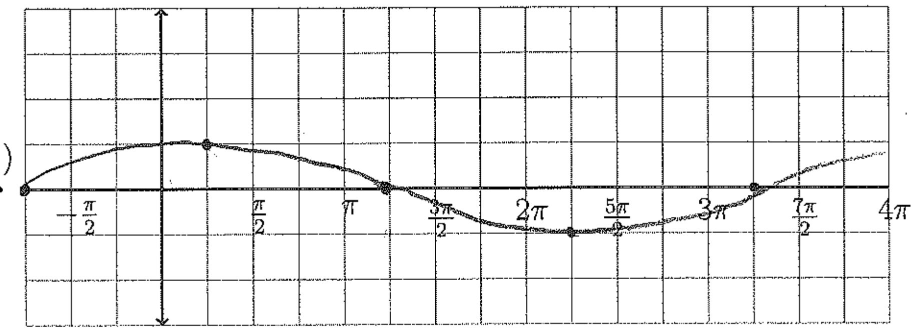
(4) Transform the graph of $f(x) = \cos x$ into the graph of $-2\cos(\frac{1}{2}x - \frac{\pi}{4}) + 1$.
First graph $y = \cos(x)$ on the grid below:



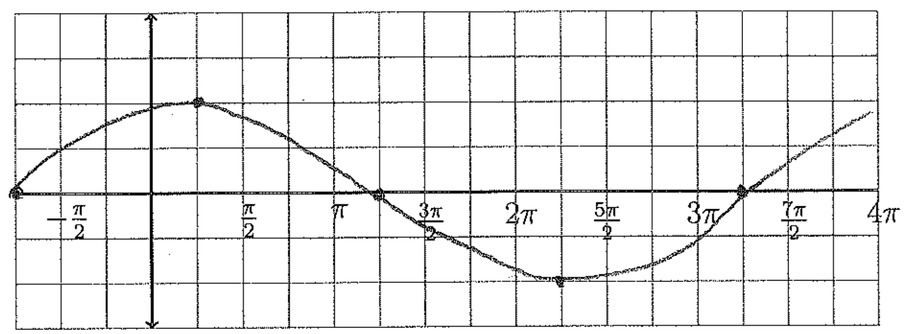
(1) $\cos(\frac{1}{2}x)$



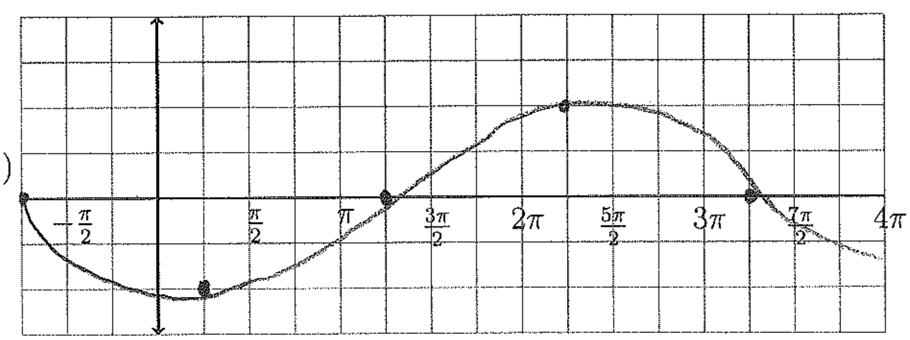
(2) $\cos(\frac{1}{2}x - \frac{\pi}{4})$



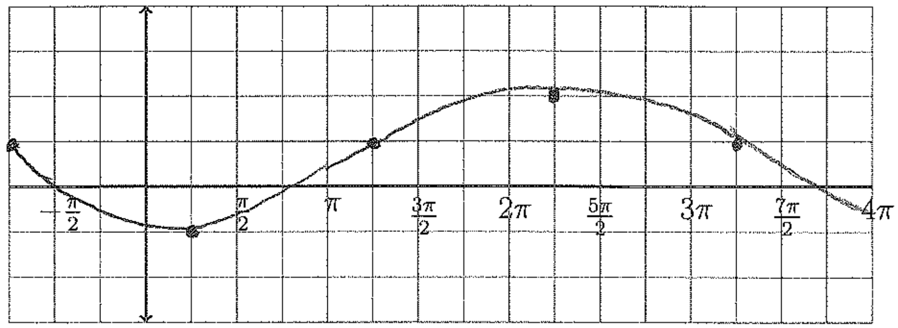
(3) $2\cos(\frac{1}{2}x - \frac{\pi}{4})$



(4) $-2\cos(\frac{1}{2}x - \frac{\pi}{4})$



(5) $-2\cos(\frac{1}{2}x - \frac{\pi}{4}) + 1$



What is the period and amplitude of the result?

period = 4π amplitude = 2

(5) Simplify the following expressions using trig identities:

(a) $\cos(2\theta) + 2\sin^2\theta = \cos^2\theta - \sin^2\theta + 2\sin^2\theta$

$= \cos^2\theta + \sin^2\theta = \boxed{1}$

(b) $\dfrac{\sin(2x)}{\cos x \sin x} = \dfrac{2\sin x \cos x}{\cos x \sin x} = \boxed{2}$

(c) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$ (hint: Don't forget to F.O.I.L.!)

$= \sin^2 x + 2\sin x\cos y + \cos^2 x + \sin^2 x - 2\cos x\sin x + \cos^2 x$

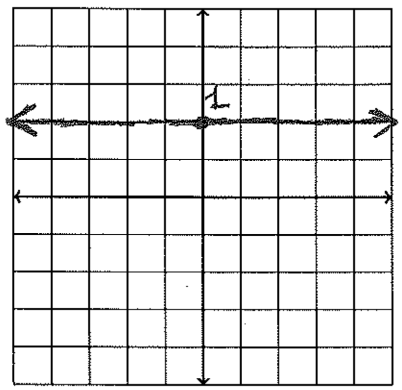
$= 2\sin^2 x + 2\cos^2 x$

$= 2(\sin^2 x + \cos^2 x = \boxed{2}$

Exponents and Logarithms

(1) In class, we drew the graph of $y = a^x$ when $a > 1$. When $a \leq 1$, we get a graph that looks different!

(a) Warm-up: What happens when $a = 1$? Draw the graph of $y = 1^x$. What is the domain and range of this function?

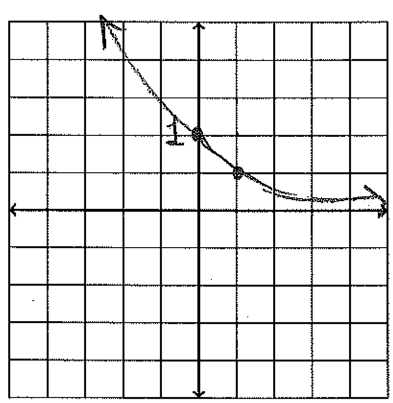


Domain: $(-\infty, \infty)$

Range: $\{1\}$

(b) Now let $a = \frac{1}{2}$. Draw the graph of $y = (\frac{1}{2})^x$. What is the domain and range of this graph? What are the biggest differences between this graph and the graph of $y = a^x$ when $a > 1$?

values of $(1/2)^x$ are larger when x is negative.



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

(c) Consider the case when $a < 0$. This function would be very difficult to graph. Why do you think that is?

$f(x) = (-a)^x$ will jump between positive and negative values.

(2) Condense the following logarithmic expressions into something that contains only one log with no coefficients (i.e. log(stuff)).

(a) $\frac{1}{2}\ln(x) + 3\ln(x+1) = \ln(x^{1/2}) + \ln(x+1)^3$

$= \ln(x^{1/2}(x+1)^3)$

(b) $2\ln(x+5) - \ln(x) = \ln(x+5)^2 - \ln(x)$

$= \ln\left(\frac{(x+5)^2}{x}\right)$

(c) $\frac{1}{3}(\log_3(x) + \log_3(x+1)) = \frac{1}{3}(\log_3(x(x+1))) = \log_3(x^{1/3}(x+1)^{1/3})$

(3) Solve the following for x:

(a) $e^{-x^2} = e^{-3x-4}$

$-x^2 = -3x-4$

$0 = x^2 - 3x - 4$

$= (x-4)(x+1)$

$\boxed{x = 4 \text{ OR } -1}$

(b) $3(2^x) = 24$

$2^x = 8$

$\log_2 8 = x$

$\log_2 2^3 = x$

$3 = x$

(c) $2(e^{3x-5}) - 5 = 11$

$2(e^{3x-5}) = 16$

$e^{3x-5} = 8$

$3x - 5 = \ln 8$

$3x = \ln 8 + 5$

$\boxed{x = \frac{\ln 8 + 5}{3}}$

(d) $\ln(3x+1) - \ln(5) = \ln(2x)$

$\ln\left(\frac{3x+1}{5}\right) = \ln(2x)$

$e^{\ln\left(\frac{3x+1}{5}\right)} = e^{\ln(2x)}$

$\frac{3x+1}{5} = 2x$

$3x + 1 = 10x$

$1 = 7x$

$\boxed{\frac{1}{7} = x}$