Euler's Method

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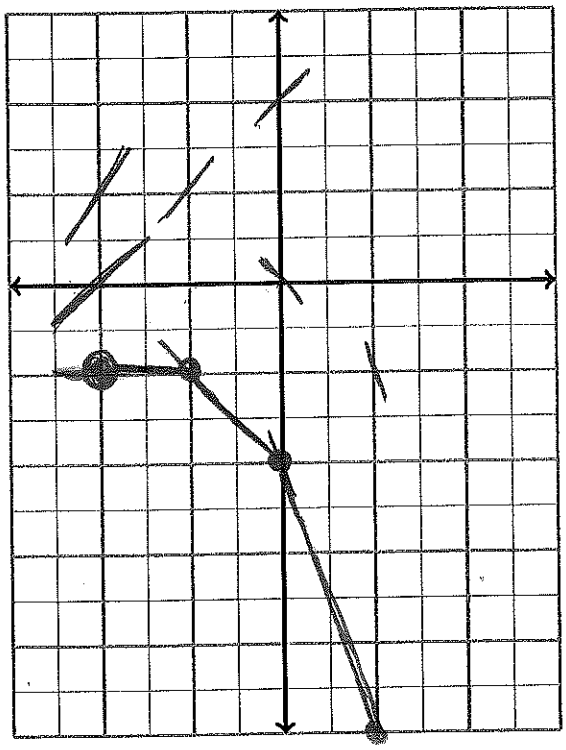
Draw part of the slope field for the differential equation $\frac{dy}{dx} = y - x - 1$.



x	y	$\frac{dy}{dx}$
-2	0	$0-(-2)-1=1$
-2	1	$1-(-2)-1=2$
-2	-1	$-1-(-2)-1=0$
-1	1	$1-(-1)-1=1$
-1	-1	$-1-(-1)-1=-1$
0	2	$2-0-1=1$
0	0	-1
0	-2	$-2-0-1=-3$
1	-1	$-1-1-1=-3$

Extra Practice

(1) Perform the following approximations (by hand):

(a) If $\frac{dy}{dx} = x + 2y$ and $y(0) = 1$, approximate $y(2)$ using $\Delta x = 1/2$.

X	Y	dy/dx	ΔY
0	1	2	$1 + 2 \cdot \frac{1}{2} = 2$
1/2	2	4.5	$2 + 4.5 \cdot \frac{1}{2} = 4.25$
1	4.25	9.5	$4.25 + 9.5 \cdot \frac{1}{2} = 9$
1.5	9	19.5	$9 + 19.5 \cdot \frac{1}{2} = 18.75$
2	18.75		

$y(2) \approx 18.75$

(b) If $\frac{dy}{dx} = xy$ and $y(1) = -1$, approximate $y(2)$ using $\Delta x = 1/2$.

X	Y	dy/dx	ΔY
1	-1	-1	$-1 + (-1)(1/2) = -1.5$
1.5	-1.5	-2.25	$-1.5 + (-2.25)(.5) = -2.625$
2	-2.625		

$y(2) \approx -2.625$

(c) If $\frac{dy}{dx} = y/x$ and $y(-2) = 1$, approximate $y(-1)$ using $\Delta x = 1/3$.

X	Y	dy/dx	ΔY
-2	1	$-1/2$	$1 + (-1/2)(1/3) = 5/6$
$-5/3$	$5/6$	$-5/10 = -1/2$	$5/6 + (-1/2)(1/3) = 4/6 = 2/3$
$-4/3$	$2/3$	$-1/2$	$2/3 + (-1/2)(1/3) = 1/2$
-1	$1/2$		

$y(-1) \approx 1/2$

Not greatest approx.

(2) Which of the three problems above can you solve exactly? Do it and compare your approximations. (b) and (c)

(b)

$\frac{dy}{dx} = xy$

$\int \frac{1}{y}\,dy = \int x\,dx$

$\ln(y) + C_1 = \frac{x^2}{2} + C_2$

$y = Ce^{x^2/2}$

$y(1) = -1$

so $-1 = Ce^{1/2}$

$C = -\frac{1}{e^{1/2}} = -e^{-1/2}$

$y = -e^{x^2/2 - 1/2}$

$y(2) = -e^{4/2 - 1/2}$

$= -e^{3/2} \approx -4.482$

(c)

$\frac{dy}{dx} = \frac{y}{x}$

$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$\ln(y) = \ln(x) + C_1$

$y = C \cdot x$

$1 = C(-2)$

$C = -\frac{1}{2}$

$y = -\frac{1}{2}x$

Notice $y(-1) = \frac{1}{2}$ exact in this one.