Final Review

1 Areas and Distances

1. Write the following expressions as a definite integral:

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \tan(3 + \frac{2i}{n})\frac{2}{n}$

   (b) $\lim_{n\to\infty} \sum_{i=1}^{n} ((\frac{5i}{n} - 1)^3 + 1)\frac{5}{n}$

   (c) $\lim_{n\to\infty} \sum_{i=1}^{n} 2^{(2+\frac{4i}{n})}\frac{4}{n}$

2. For each of the integrals and values of n below, do the following: (a) Find the left Riemann sum using n rectangles, (b) find the right Riemann sum using n rectangles, (c) find the upper Riemann sum using n rectangles, (d) use the trapezoid rule to estimate the area, (e) use Simpson's method to estimate the area.

   (a) $\int_{-1}^{1} (x-1)^2 \, dx$, $n = 4$

   (b) $\int_{-1}^{1} x^4 + 1 \, dx$, $n = 5$

   (c) $\int_{0}^{\pi/2} \cos x \, dx$, $n = 3$

2 The Definite Integral

1. (Stewart, 5.2: 35, 37, 39) Evaluate the integral by using the properties of definite integrals and by interpreting it in terms of areas.

   (a) $\int_{-1}^{2} (1-2x)dx$

   (b) $\int_{-3}^{0} (1 + 3\sqrt{9 - x^2})dx$

   (c) $\int_{-1}^{2} |x|dx$

   (d) $\int_{-4}^{4} \tan x + \sin x - 4\csc x \, dx$

2. (Stewart, 5.2: 47) Write the following as a single integral in the form $\int_a^b f(x)dx$:

   (a) $\int_{-3}^{4} 3x^7 + 2x^3 \, dx + \int_{3}^{4} \sec x \, dx$

(b) $\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx$

3. Given that $\int_0^{2\pi} f(x)dx = 4$ find the value for the following integrals:

   (a) $\int_0^{2\pi}(3f(x) + 2)\, dx$

   (b) $\int_0^{2\pi}(\frac{2f(x)}{3} + 5)\, dx$

3 The Fundamental Theorem of Calculus

1. What does the Fundamental Theorem of Calculus say? (Both parts)

2. (Stewart, 5.3: 7,9,11,12,13) Find the derivative of the given functions.

   (a) $g(x) = \int_1^x \frac{1}{t^3+1}\, dt$

   (b) $g(s) = \int_5^s (t - t^2)^8\, dt$

   (c) $F(x) = \cos x \cdot \int_x^\pi \sqrt{1 + \sec t}\, dt$

   (d) $G(x) = \int_{x^3+x}^1 \cos\sqrt{t}\, dt$

3. (Stewart, 5.3: 75) On what interval is the curve

$$y = \int_0^x \frac{t^2}{t^2 + t + 2}\, dt$$

   concave downward?

4 Indefinite Integrals

1. (Stewart, 5.4: 7, 9, 15) Find the general indefinite integral.

   (a) $\int(x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2)\, dx$

   (b) $\int(u + 4)(2u + 1)\, du$

   (c) $\int(\theta - \csc\theta\cot\theta)\, d\theta$

2. (Stewart, 5.3: 35, 37, 43) Evaluate the integral

   (a) $\int_1^2 \frac{v^3+3v^6}{v^4}\, dv$

   (b) $\int_0^1 (x^e + e^x)\, dx$

   (c) $\int_0^\pi f(x)\, dx$ where $f(x) = \sin x$ if $0 \le x < \pi/2$ and $f(x) = \cos x$ if $\pi/2 \le x \le \pi$

3. (Stewart, 5.4: 21,25,33,35,38) Evaluate the integral.

   (a) $\int_{-2}^3 (x^2 - 3)\, dx$

   (b) $\int_0^2 (2x - 3)(4x^2 + 1)\, dx$

(c) $\int_1^2 (\frac{x}{2} - \frac{2}{x})\ dx$

(d) $\int_0^1 (x^{10} + 10^x)\ dx$

(e) $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\ d\theta$

5 The Substitution Rule

1. Decide whether or not you need to use substitution for the following integrals and then evaluate them appropriately.

   (a) $\int e^{7x}\ dx$

   (b) $\int (8x^3 + 3x^2)\ dx$

   (c) $\int \cos(x/2)\ dx$

   (d) $\int e^{x^2} x\ dx$

   (e) $\int y(y^2+1)^2\ dy$

   (f) $\int_0^1 x\sqrt{1-x^2}\ dx$

   (g) $\int_1^e \frac{\ln x}{x}\ dx$

   (h) $\int_1^3 (\frac{1-x}{x})^2\ dx$

   (i) $\int_{-1}^2 \frac{x}{1+x^2}\ dx$

   (j) $\int_0^{\pi/6} \cos^3(2x)\sin(2x)\ dx$

   (k) $\int_{-1}^2 (x+2)^2\ dx$

   (l) $\int_0^2 \frac{e^x}{1+e^x}\ dx$

   (m) $\int_0^\pi \frac{\sec(3x)\tan(3x)}{\cos(3x)}\ dx$

   (n) $\int_3^4 (3-x)^{10}\ dx$

6 Areas Between Curves

1. Find the area of the region enclosed by the following curves:

   (a) $y = e^x$, $y = x^2 - 1$, $x = -1$, $x = 1$

   (b) $y = \sin x$, $y = x$, $x = \pi/2$, $x = \pi$

   (c) $y = 1/x$, $y = 1/x^2$, $x = 2$

   (d) $x = 2y^2$, $x = 4 + y^2$

   (e) $y = \sqrt{x}$, $y = \frac{1}{2}x$, $x = 9$

7 Arc Length

1. Find the arc length of $f(x) = 3x - 2$ for $-1 \leq x \leq 3$ first using Pythagorean's theorem then using the integral formula for arc length.
2. Find the length of the curve $y = 2 + 2x^{3/2}$ for $0 \leq x \leq 1$
3. Find the length of the curve $y^2 = 2(x+3)^3$ for $0 \leq x \leq 2$ and $y > 0$
4. Write the integral which expresses that arc length of $f(x) = \tan(\ln x)$ for $0 \leq x \leq \pi/4$. Don't evaluate it (gross).

8 Inverse Trig Functions

1. Evaluate the following:
   (a) $\arcsin(\pi/2)$
   (b) $\arccos(\cos(11\pi/3))$
   (c) $\arctan(\sin(-\pi/2))$
   (d) $\arcsin(14\pi/3)$
   (e) $\arctan(\tan(3\pi/4))$
2. Take the derivative of the following:
   (a) $f(x) = \arccos(2x+3)$
   (b) $g(x) = \arcsin(x) \cdot \arctan(2x)$
   (c) $h(x) = \dfrac{\arcsin(e^x)}{x}$
3. Evaluate the following integrals:
   (a) $\displaystyle\int_0^{\sqrt{2}/2} \frac{1}{\sqrt{3-x^2}}\, dx$
   (b) $\displaystyle\int_5^{5\sqrt{3}} \frac{5}{25+x^2}\, dx$
   (c) $\displaystyle\int \frac{2x}{\sqrt{1-x^2}}\, dx$