Jan 10, 2014

Galileo, Newton, & Leibniz

Galileo Galilei (1564-1642)

- 1st to clearly state that laws of nature are mathematical
- Scientific Method

Isaac Newton (1642-1727)

- invented tools of calculus
- used math to show that acceleration due to gravity is 9.8 m/s^2

-he made science make more sense!

Gottfried Wilhelm Leibniz (1646-1716)

- independently invented calc. "infinitesimal calculus"
- his notation rocked! So we now use it.

All calc fathers ganged up on Leibniz, saying Newton invented calculus first.

Newton's Question:

How do we find the velocity of a moving object at time t?

- tough parts: what do we mean by velocity of an object at the instant of time t?

Average Velocity (not the answer to Newton's Question) but a first step

The Ball Drop

- At time t, how far has the ball fallen? Just measure it!
- How fast is the ball falling at time t? Harder..

DEF: The <u>average velocity</u> from $t = t_1$ to $t = t_2$ is

$$\text{avg velocity} = \frac{\text{change in distance}}{\text{change in time}}$$

= Slope of the secant line which goes through two data pts

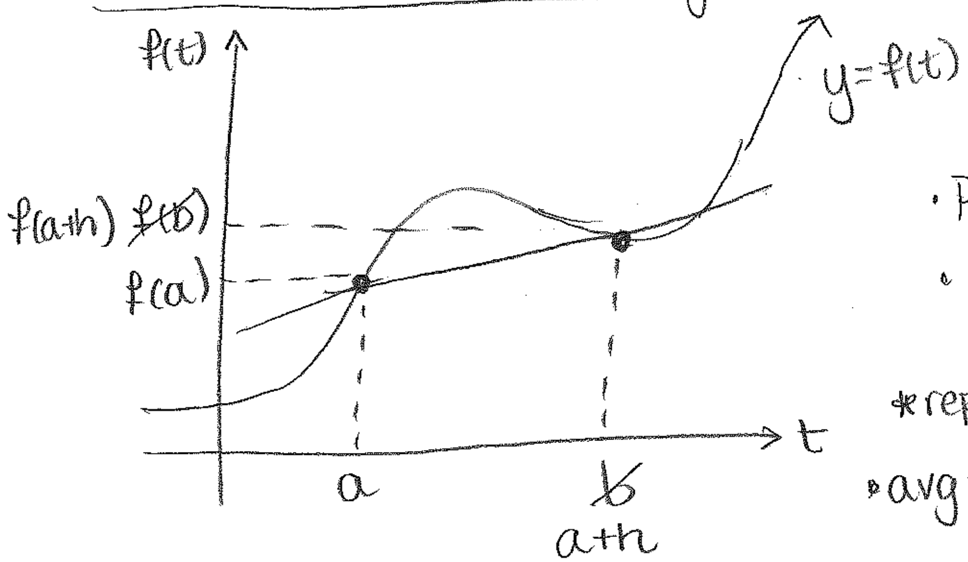
(similarly, the average acceleration from $t = t_1$ to $t = t_2$ is

$$\text{avg acc} = \frac{\text{change in velocity}}{\text{change in time}}$$

*Ball Drop - Fill in Numbers

*WeBWork quirk

A General Look at average velocity



- Pick 2 pts on curve
- Avg vel $= m = \dfrac{f(b) - f(a)}{b - a}$

*replace b with a+h

- avg vel $= m = \dfrac{f(a+h) - f(a)}{a+h-a}$

$$= \frac{f(a+h) - f(a)}{h}$$

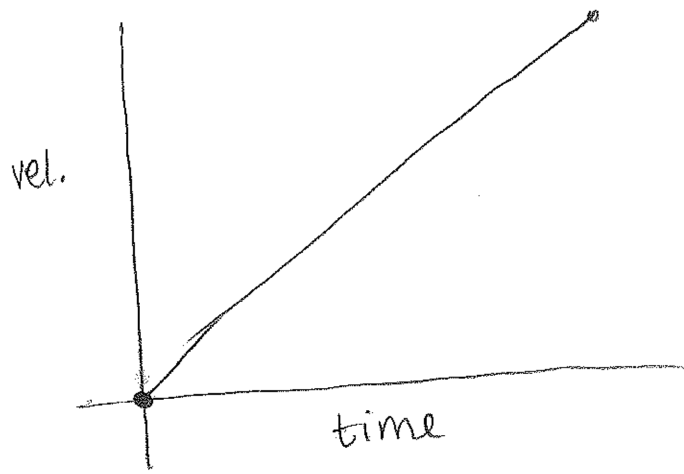
*the smaller h is, the closer we get to this notion of "instantaneous velocity" but we can't divide by zero!

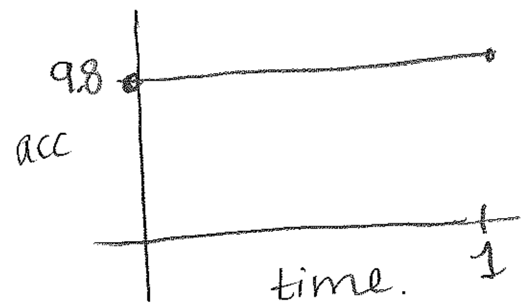
*Ball Drop wksht.

avg. acc. is constant

$\Rightarrow$ secant lines going through two pts on velocity graph always has the same slope

$\Rightarrow$ velocity graph falls in a straight line!





$V(0)=0$, slope $= 9.8$

$V(t) = 9.8t$

Can we get a better estimate for position function?

- Galileo: If velocity is linear,

$$f(t) = \frac{1}{2} \cdot V(t) \cdot t$$

So $\underbrace{f(t)}_{\text{position function}} = \frac{1}{2}(9.8t)t = 4.9t^2$

So, <u>this time</u>, we didn't need instantaneous vel.
but we will...

Goal: Rates of change in general

$f(x)$ could be: distance vs. time x
profit vs. supply x
birthrate vs. population x ....

<u>DEF</u>: Given fnctn f, the <u>avg rate of change</u> of f over an interval $[x, x+h]$ is

$$\frac{f(x+h)-f(x)}{h}$$

> the <u>instantaneous rate of change</u> of f at a pt x is the limit of the avg rates of change $[x, x+h]$ as $h \to 0$

this is the beginning of calculus!

Why do mathematicians love e^x?

p. 81

this table leads us to believe that the rate of change of $f(x)=e^x$ at an x is e^x.