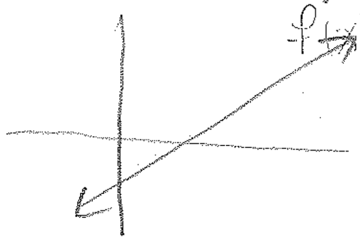
Jan. 6, 2014

A Functional Review of Functions

A function is a rule:

$$x \mapsto \overset{\text{Rule}}{\boxed{f}} \mapsto f(x)$$

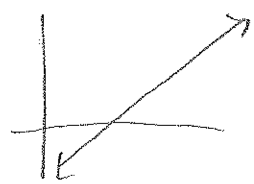
Simplest Function: line



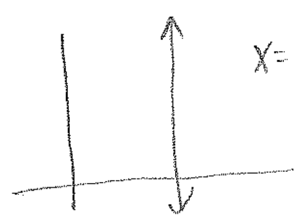
Domain of $f(x)$: Values of x for which $f(x)$ is defined, (in our case also a real #)

Range of $f(x)$: Values of y which satisfy $y=f(x)$ for some x.

Ex: For a line D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

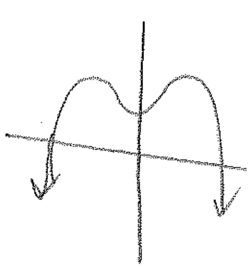


<u>unless</u>... line is vertical $(x=c)$
D: $\{c\}$
R: $(-\infty, \infty)$

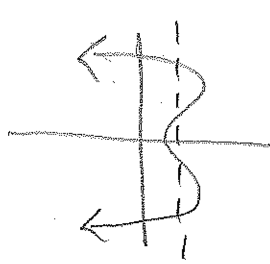


* A graph is the graph of a function if every x value is mapped to at most one y value (vertical line test)

Function

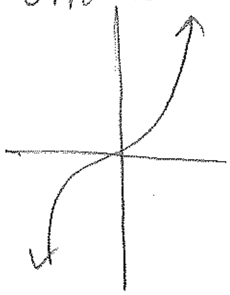


Not a function

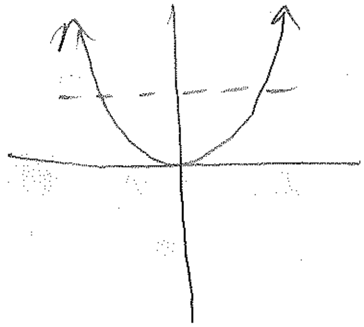


DEF: A function is one-to-one if for every y, there is at most one x which maps to that y (horizontal line test)

one-to-one



not one-to-one

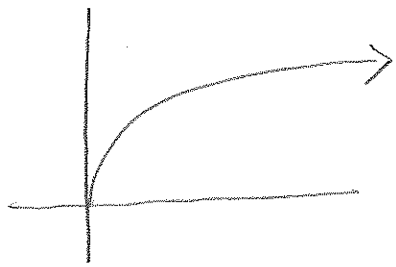


Some Functions

- $f(x) = \sqrt{x}$

  D: $x \geq 0 \Longleftrightarrow [0,\infty)$

  R: $x \geq 0 \Longleftrightarrow [0,\infty)$



* Interval notation: (write all the value x can be)

$a \leq x \leq b \Longleftrightarrow [a,b]$

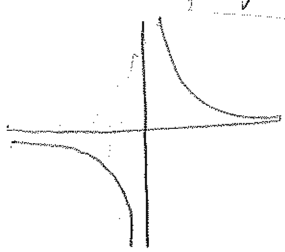
$a < x \leq b \Longleftrightarrow (a,b]$

$a \leq x \Longleftrightarrow [a,\infty)$

all real #'s $\Longleftrightarrow (-\infty,\infty)$

$a \leq x < b$ OR $c < x \leq d \Longleftrightarrow [a,b) \cup (c,d]$

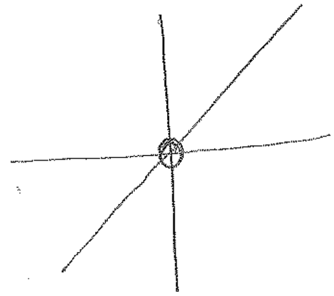
- $g(x) = 1/x$



  D: $x \neq 0 \Leftrightarrow (-\infty,0) \cup (0,\infty)$

  R: $x \neq 0 \Leftrightarrow (-\infty,0) \cup (0,\infty)$

∘ $h(x)=\frac{x^2}{x}=x$

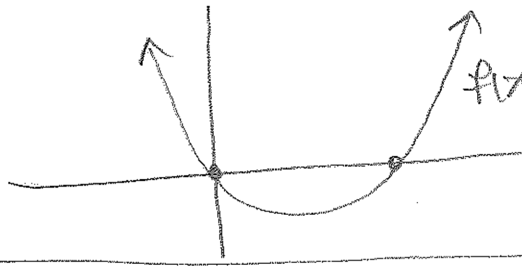


D: $x\neq 0 \Leftrightarrow (-\infty,0)\cup(0,\infty)$

R: $x\neq 0 \Leftrightarrow (-\infty,0)\cup(0,\infty)$

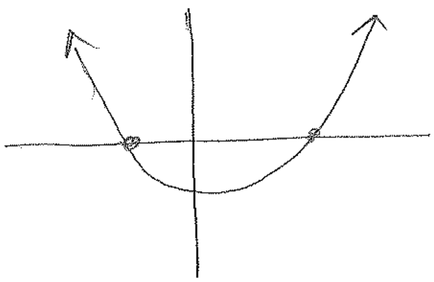
∘ Review basic functions on pg. 3

New Functions From Old

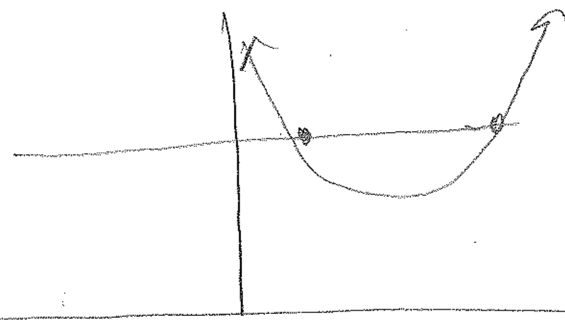


Left/Right Shift

Left shift: $f(x+2)$

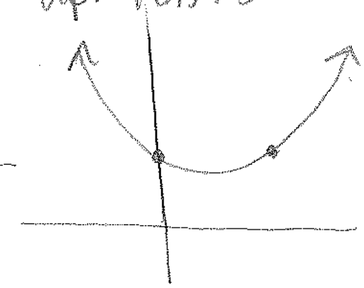


Right: $f(x-1)$

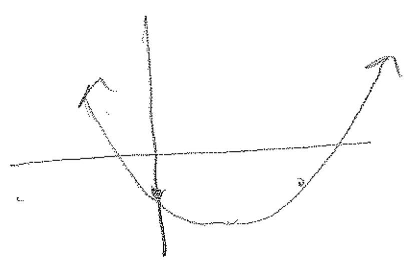


Up/Down Shift

Up: $f(x)+3$

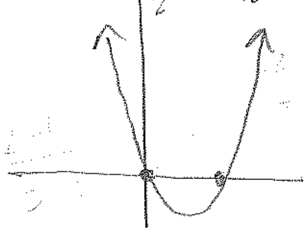


Down: $f(x)-2$

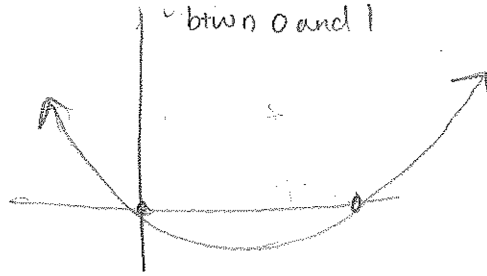


Horizontal Squeeze/Expand

Squeeze: $f(\underbrace{3}_{\text{bigger than } 1}x)$

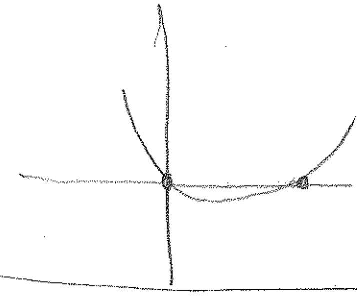


Expand: $f(\underbrace{\tfrac{1}{2}}_{\text{btwn 0 and 1}}x)$

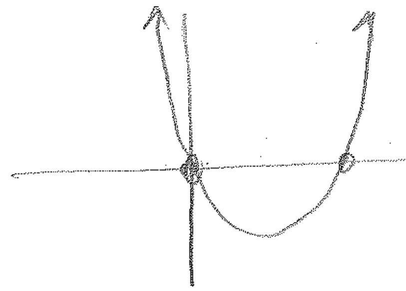


Vertical Squeeze/ Expand

Squeeze: $y=\frac{1}{3}f(x)$

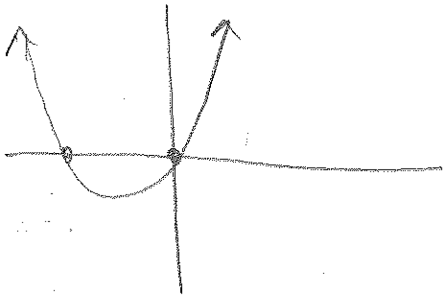


expand: $y=4f(x)$



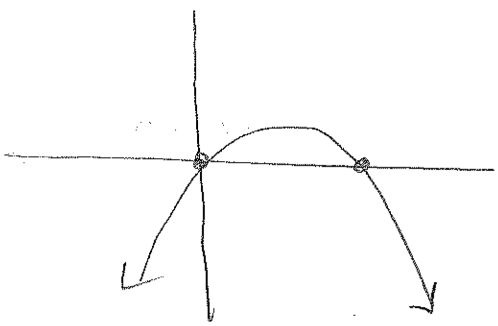
Vertical Reflection

$y=f(-x)$



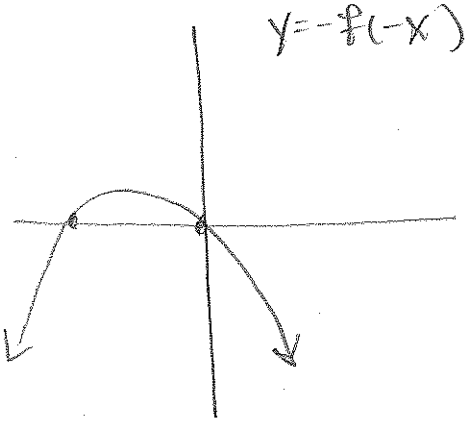
horizontal reflection

$y=-f(x)$



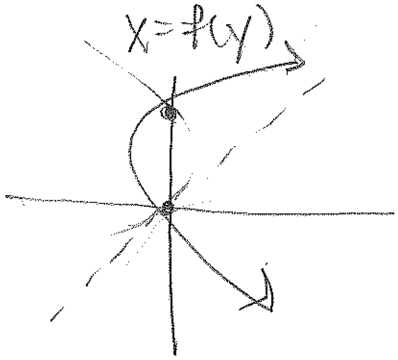
180° rotation

$y=-f(-x)$



flip over line $x=y$

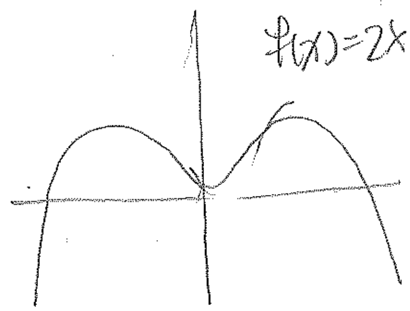
$x=f(y)$



* Practice: Pg. 8 of Handout

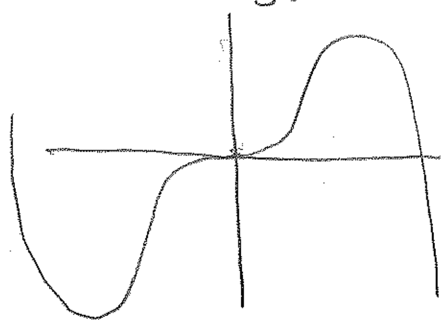
Symmetries

$f(x) = 2x^2 - x^4$



A function is <u>even</u> if it satisfies $f(-x) = f(x)$ (has symmetry across the y-axis)

$f(x) = 2x^3 - x^5$



A function is <u>odd</u> if it satisfies $f(-x) = f(x)$ (has 180° rotational symmetry)

<u>Ex</u>: Is $f(x) = \dfrac{x^3+x}{x+\frac{1}{x}}$ even, odd, or neither?

$$f(-x) = \frac{(-x)^3 - x}{-x - \frac{1}{x}} = \frac{-(x^3) - x}{-x - \frac{1}{x}} = \frac{-(x^3+x)}{-(x+\frac{1}{x})} = \frac{x^3+x}{x+\frac{1}{x}} = f(x)$$

$\boxed{\text{even}}$

Composition

$f(x), g(x)$

$(f \circ g)(x) = f(g(x))$

<u>Ex</u>: $f(x) = x^2 - 4$, $g(x) = x - 2$

$(f \circ g)(x) = (g(x))^2 - 4 = (x-2)^2 - 4$

Inverse Function

- $f(x)$ is 1-1 funct (hor. line test)
- $g(x)$ is a funct satisfying $f(g(x)) = g(f(x)) = x$

then g is the inverse function of f. Write $g(x) = f^{-1}(x)$

Ex: $f(x) = x^3$, $f^{-1}(x) = \sqrt[3]{x}$

$f(f^{-1}(x)) = (\sqrt[3]{x})^3 = x$

$f^{-1}(f(x)) = \sqrt[3]{(x^3)} = x$

How to find inverse of $f(x)$:

(1) Set $x = f(y)$

(2) Solve for y

(3) $y = f^{-1}(x)$

Ex: $f(x) = \frac{2x}{x-1}$

$x = \frac{2y}{y-1}$

$y = \frac{-x}{2-x} = f^{-1}(x)$

★ To get the graph of $f^{-1}(x)$, flip graph over line $y=x$