Hour Exam #1

Math 3

February 4, 2014

Name (Print): KEY
Last First

On this, the first of the two Math 3 hour-long exams in Winter 2014, and on the second hour-long exam, and on the final exam, I will work individually, neither giving nor receiving help, guided by the Dartmouth Academic Honor Principle.

Signature:______________________________

Section (circle):

10 (Diesel) 11 (Martinez)

Instructions: You are not allowed to use calculators, books, or notes of any kind. All of your answers must be marked on the Scantron form provided or entered on the test, depending on the problem. Take a moment now to print your name and section clearly on your Scantron form and on page 1 of your exam booklet and sign the affirmation. You may write on the exam, but you will only receive credit on Scantron (multiple-choice) problems for what you write on the Scantron form. At the end of the exam, you must turn in both your Scantron form and your exam booklet. There are 13 multiple-choice problems worth 4 points each and 3 long-answer written problems worth a total of 52 points.

Problem	Points	Score
14	11	
15	16	
16	21	
MC	52	
Total	100	

On this page, let $f(x) = \dfrac{6x+2}{2x^2-3x+1}$.

1. What is $\lim_{x\to\infty} f(x)$?

   (a) $\frac{1}{3}$

   (b) ∞

   (c) $-\infty$

   (d) 0

   (e) 3

$$\lim_{x\to\infty} \frac{6x+2}{2x^2-3x+1} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{\cancel{6/x} + \cancel{2/x^2}}{2 - \cancel{3/x} + \cancel{1/x^2}} = 0$$

2. What is the domain of $f(x)$?

   (a) $(-\infty, \infty)$

   (b) $(-\infty, 1) \cup (1, \infty)$

   (c) $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 1) \cup (1, \infty)$

   (d) $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

   (e) $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, 1) \cup (1, \infty)$

$2x^2 - 3x + 1 \neq 0$

$(2x-1)(x-1) \neq 0$

$x \neq 1/2, 1$

3. Let $f(x) = \frac{1}{2x+1}$. What is $f^{-1}(x)$?

(a) $\frac{1}{\frac{2}{x}+1}$

(b) $2x+1$

(c) $\frac{\frac{1}{x}-1}{2}$

(d) $-\frac{x}{2}$

(e) $\frac{1}{2x-2}$

$x = \frac{1}{2y+1}$

$x(2y+1) = 1$

$2y+1 = \frac{1}{x}$

$2y = \frac{1}{x} - 1$

$y = \frac{\frac{1}{x} - 1}{2}$

4. $\lim_{x \to -1} \frac{1}{x+1} =$

(a) 1

(b) ∞

(c) $-\infty$

(d) 0

(e) This limit does not exist.

$\lim_{x \to (-1)^-} \frac{1}{x+1} = -\infty$

$\lim_{x \to (-1)^+} \frac{1}{x+1} = \infty$

$\lim_{x \to -1} \frac{1}{x+1}$ DNE

5. Find the derivative of $\sin(\tan(x^2))$.

(a) $\cos(\tan(x^2)) \cdot \sec^2(x^2) \cdot 2x$
(b) $\sin(\sec^2(x^2)) \cdot \cos(x^2) \cdot 2x$
(c) $-\cos(\sec^2(2x))$
(d) $\sin(\tan(2x)) \cdot \sec^2(2x)$
(e) $\cos(\tan(x^2)) \cdot \sec^2(2x) \cdot 2x$

$\frac{d}{dx} \sin(\tan(x^2))$
$= \cos(\tan(x^2)) \cdot \sec^2(x^2) \cdot 2x$

6. Given $x^2y + y^2x = 20$, find $\frac{dy}{dx}$ at $(4, 1)$.

(a) $-\frac{9}{8}$
(b) -1
(c) $-\frac{9}{24}$
(d) 0
(e) The derivative is undefined at $(4, 1)$.

$\frac{d}{dx}(x^2y + y^2x) = \frac{d}{dx} 20$
$2xy + x^2\frac{dy}{dx} + 2yx\frac{dy}{dx} + y^2 = 0$
$x^2\frac{dy}{dx} + 2yx\frac{dy}{dx} = -y^2 - 2xy$
$\frac{dy}{dx} = \frac{-y^2 - 2xy}{x^2 + 2yx}$
$x = 4, y = 1$
$\left.\frac{dy}{dx}\right|_{(4,1)} = \frac{-(1)^2 - 2\cdot4\cdot1}{4^2 + 2\cdot1\cdot4}$
$= \frac{-9}{24}$

7. Assume $f(x)$ is a function that is continuous on $[0, 10]$ and differentiable on $(0, 10)$. If $f(0) = 35$ and $f(10) = 5$, which of the following must be true?

(a) $f(x)$ is a decreasing function.
(b) $f'(x) = -3$ for all x where $0 < x < 10$.
(c) $f'(x) = -3$ for at least one value of x between 0 and 10.
(d) $f(x) = 0$ for at least one value of x between 0 and 10.

MVT:
avg rate of change $= \frac{5 - 35}{10 - 0} = -3$

by mean value thm.

8. Let $p(t) = 3t^2 - 6t$ give the position at time t of an object moving left and right along an axis, with left = negative direction and right = positive direction. Find the velocity of the object and the direction of movement when $t = 1$.

(a) velocity = -1 units/sec, moving right

(b) velocity = -3 units/sec, moving left

(c) velocity = 0 units/sec, stationary

(d) velocity = -2 units/sec, moving left

(e) None of the above.

$p'(t) = v(t) = 6t - 6$

$v(1) = 6(1) - 6 = 0$

9. Let $y = \dfrac{\ln(e^{\pi x})}{\pi^2}$. Find y'.

(a) $\dfrac{1}{\pi}$

(b) $\dfrac{\pi - 2}{\pi^2}$

(c) $\dfrac{1}{\pi^2 e^{\pi x}}$

(d) $\dfrac{\pi x - 1}{\pi^2} \cdot \ln(e^{\pi x - 1})$

(e) $\dfrac{\ln \pi}{e^{\pi x}}$

$y = \dfrac{\ln(e^{\pi x})}{\pi^2} = \dfrac{1}{\pi^2} \ln(e^{\pi x})$

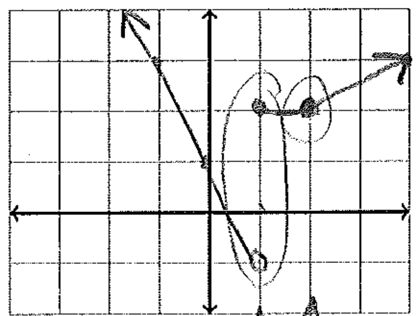
$= \dfrac{1}{\pi^2}(\pi x)$

$= \dfrac{1}{\pi} \cdot x$

$y' = \dfrac{1}{\pi}$

For this page, let $f(x) = \begin{cases} -2x+1, & \text{if } x < 1 \\ 2, & \text{if } 1 \leq x < 2 \\ \frac{1}{2}x+1, & \text{if } 2 \leq x \end{cases}$

Graph $f(x)$ on the provided axes (this will not be graded).



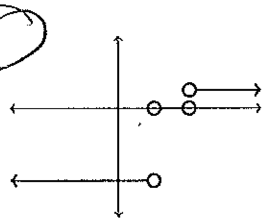
10. Which of the following is <u>true</u> about $f(x)$?

~~(a)~~ $f(x)$ is continuous for all real numbers
~~(b)~~ $f(x)$ is differentiable for all real numbers
(c) $f(x)$ has a removable discontinuity at 2
(d) $f'(-1) = -2$
(e) None of the above.

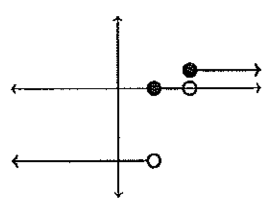
Not diff'ble so the derivative will have holes in its graph

11. Which of the following could be the graph of $f'(x)$?

(a)

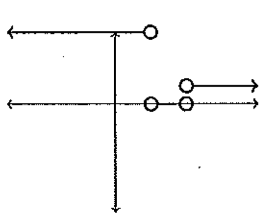


(b)

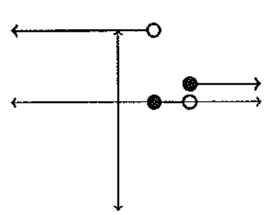


(e) None of the above.

(c)



(d)



12. Use Newton's Method to estimate a solution to the equation $x^2 + x - 3 = 0$. Use $x_0 = 1$ and give your result for x_2.

(a) $x_2 = \frac{4}{3}$

(b) $x_2 = \frac{43}{33}$

(c) $x_2 = \frac{-1+\sqrt{13}}{2}$

(d) $x_2 = -\frac{1}{3}$

(e) None of the above.

i	x_i	$f(x_i)$	$f'(x_i)$	$x_i - \frac{f(x_i)}{f'(x_i)}$
0	1	-1	3	$1 - \frac{-1}{3} = 4/3$
1	4/3	1/9	11/9	$4/3 - \frac{1/9}{33/9} = \frac{44}{33} - \frac{1}{33} = \boxed{\frac{43}{33}}$

$= X_2$

$f(x) = x^2 + x - 3$

$f'(x) = 2x + 1$

$f(4/3) = \frac{16}{9} + \frac{12}{9} - \frac{27}{9} = \frac{1}{9}$

$f'(4/3) = 8/3 + 3/3 = 11/3 = 33/9$

13. Find the linearization of $c(t) = \frac{\sqrt{t+2}}{4}$ at the point $(2, \frac{1}{2})$ and use it to approximate $c(2.1)$.

(a) 2

(b) $\frac{81}{160}$

(c) $\frac{\sqrt{4.1}}{4}$

(d) $\frac{79}{160}$

(e) $\frac{9}{16}$

$c'(t) = \frac{1}{4} \cdot \frac{1}{2\sqrt{t+2}}$

$c'(2) = \frac{1}{4} \cdot \frac{1}{2\sqrt{2+2}} = \frac{1}{16}$

linearization @ $(2, 1/2)$:

$y - \frac{1}{2} = \frac{1}{16}(x-2) \Rightarrow y = \frac{1}{2} + \frac{1}{16}(x-2)$

approximate $c(2.1)$:

$\frac{1}{2} + \frac{1}{16}(2.1-2) = \frac{1}{2} + \frac{1}{16} \cdot \frac{1}{10} = \frac{1}{2} + \frac{1}{160}$

$= \frac{80}{160} + \frac{1}{160}$

$\boxed{= \frac{81}{160}}$

Out of 11

Long answer questions

14. Find the equation of the line tangent to the function $f(x) = 2x^3 + x - 1$ at $x = -1$. Show all your work.

Slope of tangent: $f'(x) = 6x^2 + 1$

Slope at $x=-1$: $f'(-1) = 6(-1)^2 + 1$
$= 7$

Equation of tangent through $(-1, f(-1)) = (-1, -4)$: $y+4 = 7(x+1)$

some other forms:

$y = 7x + 3$

$y + 4 = 7x + 7$

16 pts

15. Let $f(x) = 2x^2 + x + 3$.

(a) State the limit definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Calculate the derivative of $f(x)$ using the limit definition of the derivative. Show all your work and explain your steps to receive any credit.

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + (x+h) + 3 - 2x^2 - x - 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{x} + h + \cancel{3} - \cancel{2x^2} - \cancel{x} - \cancel{3}}{h}$$

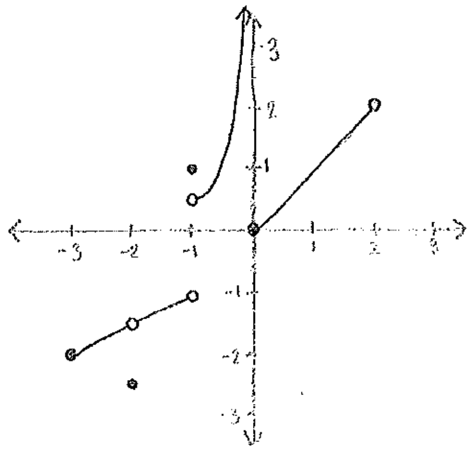
$$= \lim_{h \to 0} \frac{4xh + 2h^2 + h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 1)}{\cancel{h}}$$

$$= \lim_{h \to 0} 4x + 2h + 1$$

$$= \boxed{4x + 1}$$

16. Consider the (beautifully drawn) graph of $f(x)$ below:



(a) What is the domain of $f(x)$?

$[-3, 2)$

(b) For what values of x is $f(x)$ discontinuous on its domain?

discontinuous at $x = -2, -1, 0$.

$\lim_{x \to (-2)^-} f(x) = -3/2 = \lim_{x \to (-2)^+} f(x)$ but $f(-2) = -5/2$

$\lim_{x \to (-1)^-} f(x) = -1$, $\lim_{x \to (-1)^+} f(x) = 1/2$ $\neq$

$\lim_{x \to 0^-} = \infty$, $\lim_{x \to 0^+} = 0$ $\neq$

(c) Evaluate the following limits (or state that they do not exist):

$\lim_{x \to 0^-} f(x) = \infty$

$\lim_{x \to 0} f(x) =$ DNE

$\lim_{x \to (-1)^-} x \cdot f(x) = \lim_{x \to (-1)^-} x \cdot \lim_{x \to (-1)^-} f(x) = (-1) \cdot (-1) = \underline{1}$

(d) Where does f have removable discontinuities? List the x values for these discontinuities together with the corresponding y value that would remove the discontinuity there.

Removable Discontinuity only at $x = -2$.

$\lim_{x \to (-2)^-} f(x) = -\frac{3}{2} = \lim_{x \to (-2)^+} f(x)$

so define $y = -3/2$ when $x = -2$.