1. Position, Velocity, Acceleration

(1) A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.

(2) A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?

2. Related Rates

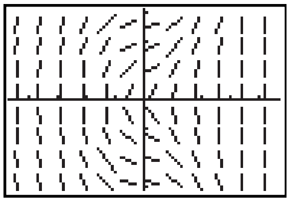
(1) The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square.

(2) The bottom of a rectangular swimming pool is 25×40 meters. Water is pumped into the tank at the rate of 500 cubic meters per minute. Find the rate at which the level of the water in the tank is rising.

(3) Find the rate of change of the volume of a cylinder of radius r and height h with respect to a change in the radius, assuming the height is also a function of r. (Note that this is with respect to radius, not time)

3. Slope Fields

Match the slope fields to their corresponding differential equations.

1.


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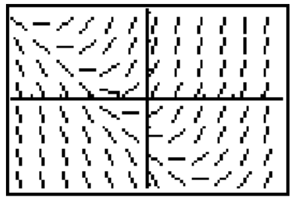

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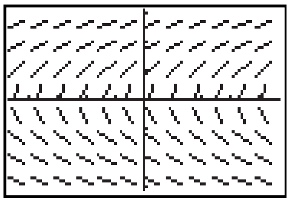

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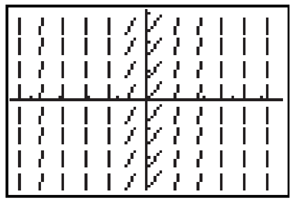

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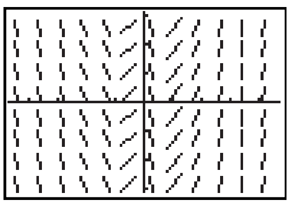

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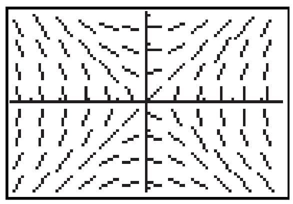

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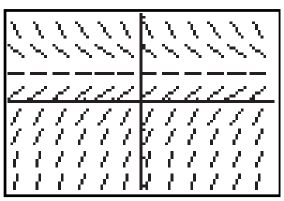

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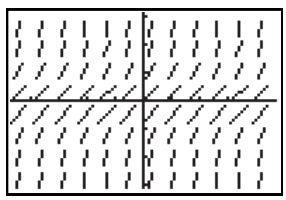

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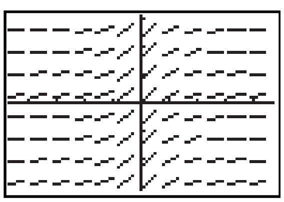

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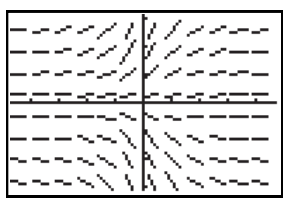

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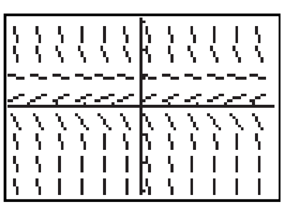

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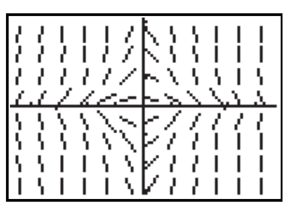

Part II

A $\frac{dy}{dx} = 2y(1-y)$

B $\frac{dy}{dx} = 1-y$

C $\frac{dy}{dx} = 3x^2 + 1$

D $\frac{dy}{dx} = \frac{x}{y}$

E $\frac{dy}{dx} = {}^{-}2xy$

F $\frac{dy}{dx} = x + y + 1$

G $\frac{dy}{dx} = \frac{1}{y}$

H $\frac{dy}{dx} = \frac{1}{1+x^2}$

I $\frac{dy}{dx} = \frac{y}{1+x^2}$

J $\frac{dy}{dx} = 2x - \frac{1}{x}$

K $\frac{dy}{dx} = \frac{1+3x^2}{2y}$

L $\frac{dy}{dx} = 1 + y^2$

4. Exponential Growth and Decay

(1) A biologist is researching a newly-discovered species of bacteria. At time $t = 0$ hours, he puts one hundred bacteria into what he has determined to be a favorable growth medium. Six hours later, he measures 450 bacteria. Given that the rate of population growth is proportional to the population, how many bacteria will there be after 10 hours?

(2) The same biologist is performing an experiment, but with different bacteria. Unfortunately, he can't remember how many bacteria he started with (FAIL). He does know that there are 200 bacteria after 1 hour and 500 bacteria after 2 hours. How many bacteria did he start with? Assume the rate of population growth is proportional to the population.

(3) A lake of water is at a temperature of 60 degrees F. The air temperature drops to 30 degrees F. Assume that rate of cooling is proportional to the difference between the water temp and the air temp. If the water temperature drops 10 degrees F during the first day, how long will it take till the temperature of the lake is 40 degrees F?

(4) Secret organism "Q" doubles in size every 6 hours, how long does it take to get 100 times as big?

5. Euler's Method

(1) (a) Use Euler's method with step size 0.2 to estimate $y(0.4)$, where $y(x)$ is the solution of the initial-value problem $\frac{dy}{dx} = x + y^2$, $y(0) = 0$.

(b) Repeat part (a) with step size 0.1.

(2) Use Euler's method with step size 0.5 to approximate $y(4)$ where $y(x)$ is the solution of the initial-value problem $\frac{dy}{dx} = y + 2x$, $y(1) = 0$.

6. Issues in Curve Sketching

For each of the following graphing problems also determine (a) where $f(x)$ is defined, (b) where $f(x)$ is continuous, (c) where $f(x)$ is differentiable, (d) where $f(x)$ has vertical asymptotes, (e) if $f(x)$ has horizontal asymptotes, (f) where $f(x)$ is increasing and where it is decreasing, (g) where $f(x)$ is concave up and where it is concave down, (h) what the critical points of $f(x)$ are, and (i) where the points of inflection are. Then sketch a graph.

(1) Graph $f(x) = x^5 + -4x^4 + 4x^3$.

(2) Graph $f(x) = \dfrac{x^3}{x^2+1}$.

(3) Graph $f(x) = (x-2)^2(x-1)$.

(4) Graph $f(x) = x + 1/x$.

7. Optimization

(1) Find the dimensions of the rectangle of area 96 cm^2 which has minimum perimeter. What is this minimum perimeter?

(2) An open box (meaning there is no lid) is to be made out of a given quantity of cardboard of area 9cm. Find the maximum volume of the box if its base is square.

(3) An enemy jet is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). At what point will the jet be at when the soldier and the jet are closest? (this one is tricky. Think about using Pythagorean theorem to measure distance between the two objects)